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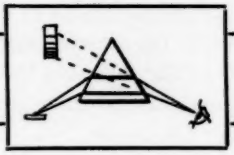

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PLANE WAVES OF LIGHT III

ABSORPTION BY METALS

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[Bell Telephone Laboratories, New York, N. Y. Received March 15, 1932]

§21. *Introduction:* The study which follows has two different aspects. In one sense it is a continuation of two earlier papers[1] bearing the same general title; and as we shall make many cross-references it has seemed wise to run the reference numbers of the present paper consecutively with the former ones. In another sense it is a detail of a more general investigation, the purpose of which is to study the relationship between the optical and photoelectric properties of metals. We begin by indicating its connection with this latter study.

On the experimental side, the general investigation makes much use of thin films of alkali metal deposited either on glass or on a metallic base, frequently platinum. The study of such films, rather than bulk materials, recommends itself for three reasons: first, because they show remarkable variations in photo-sensitivity as the plane of polarization and angle of incidence of the light is varied; second, because the photoelectrons released from bulk material by ordinary light must originate in a very thin layer near the surface, since otherwise they would soon lose their energy by collision; and third, because by working with bases

[1] Plane Waves of Light, I, J.O.S.A. *15*, 137–161; 1927; and II, J.O.S.A. *16*, 1–25; 1928

having widely different optical properties, we can distinguish the effects which are due to the photo-sensitive film itself from those due to the nature of the base. This we cannot do if bulk materials are used, though the base (the underlying bulk material) is always present as a possible disturbing factor.

To interpret such experiments adequately, however, we need complete knowledge of the energy distribution within the photo-sensitive layer, and of its relation to the optical properties of the base. This is a purely optical problem, capable of mathematical solution when the optical constants of the materials are known. It is the mathematical study of this optical problem, and in particular of the amount of energy absorbed by the film, with which the present paper deals.

§22. *Physical Background:* The following broad statement of physical background may serve to motivate our mathematical presentation:

When a beam of light shines upon a metallic plate, the electric intensity inside the plate is proportional to the intensity in the incident beam. The factor of proportionality, however, is not an *absolute* constant. This is obvious since it differs for different metals, and depends on the medium (air or glass, for example) through which the light reaches it. It is not even an *intrinsic* constant of the metal, for if the intensity of the incident beam is kept fixed the electric intensity inside the metal will be different for different planes of polarization, or for different angles of incidence, or for different wave lengths. Moreover, even if the intensity, color and plane of polarization of the incident light are all kept fixed, and only the angle of incidence is allowed to change, the curves which relate the intensity inside the metal to the angle of incidence will be of different shapes at different depths below the surface, due to the fact that the light is extinguished more rapidly the more oblique its angle of incidence. This last point is illustrated by Fig. 21, in which the electric intensity is plotted against angle of incidence at various depths below the surface of a plate of rubidium.

The rate at which energy is absorbed by a metal is, according to the Maxwell theory, directly proportional to the electric intensity *at the place where the absorption takes place;* that is, inside the metal. The constant of proportionality is, in this case, an intrinsic property of the metal. That is, it does not vary with the plane of polarization or angle of incidence of the incident light, nor does it vary from place to place within the metal. It may vary from wave length to wave length, but only if the optical constants also vary.

It must be emphasized that this "*intrinsic absorbing power*," as we shall hereafter call it, is based upon the electric intensity *inside* the metal. It is not what is commonly called the "absorbing power" of the metal. The latter is the ratio of the rate of absorption to the intensity of the *incident* beam, and contains not only the intrinsic factor, but also the other factor of proportionality to which we referred in the first paragraph. It therefore varies with almost every conceivable change in the incident beam. As we shall see in §24, this common "absorbing

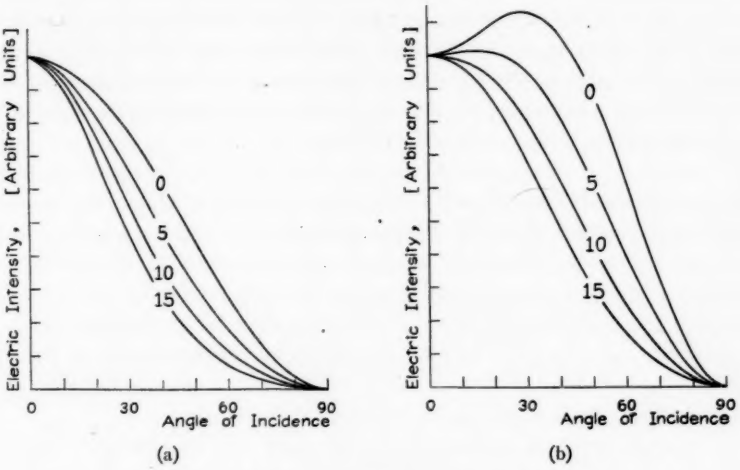

FIG. 21. ELECTRIC INTENSITY VS. ANGLE OF INCIDENCE

The electric vector in (a) is normal to, and in (b) parallel to, the plane of incidence. The optical constants used are those of rubidium at $\Lambda_0=4893$ *A:* $N=0.134$, $K_0=0.868$. *The numbers on the curves denote depths (in cm* $\times 10^{-6}$) *below the surface of the plate.*

power" is not primarily governed by the ability of a substance to absorb energy. In fact, it might more appropriately be called a "refracting power," particularly since it depends upon the material through which the light arrives,[2] as well as that in which absorption takes place. However, in order not to depart too widely from established usage, we shall call it the "bulk absorbing power" of the material, expanding this phrase occasionally to such forms as the "bulk absorbing power of platinum from air," or " . . . from glass," when it is necessary to call particular attention to the medium of incidence.

[2] The term "absorbing power" usually implies that the incident beam is in free space.

It does not seem unreasonable to assume that the relationship between the rate at which a metal absorbs light energy and the rate at which it gives off photoelectrons is also an intrinsic property of the metal: that is, that it does not depend at all upon plane of polarization or angle of incidence and that its relation to wave length is not a very complicated one. This hypothesis has neither been proved nor disproved. It merely belongs to the class of things which, if experimentally demonstrated, would be characterized as "what was to have been expected." If it is true, the highly complicated way in which photo-emission varies with color, plane of polarization and angle of incidence must be attributed primarily to the fact that the absorbing power of the metal varies with all these factors, not to any peculiarities of the mechanism of photo-emission. It is even possible that part of the variation of emissivity with wave length may be capable of a similar explanation.

On the other hand, even if the hypothesis is false, the ratio of photo-emission to incident intensity is still the product of three factors: one having to do with the refractive peculiarities of the metal, a second with its intrinsic absorbing power, and only the third with the mechanism of photo-emission per se. Before we can effectively study the third, therefore, it seems highly desirable to determine the first two independently; by doing this, we shall be able to correct our observations for purely optical influences, and thus arrive at results which can properly be said to represent the *intrinsic photo-emissivity of the metal.*

Such a program is being carried out in the Bell Laboratories by Dr. H. E. Ives and his associates. Some of the experimental results have already been published,[3] but the study has so far been handicapped by the lack of reliable optical constants in the spectral region where the work should be done, and by the need for an adequate theory of the purely optical effects. As was said above, the present paper aims to supply the latter need.

For simplicity, we have so far spoken as if we were to deal with only two media: But as the experimental set-up involves three media more often than two, our mathematical study must also include three media: the slab or film in which we are principally interested, the base upon which it rests, and the medium through which the light reaches it. Naturally, the presence of the base does not affect the "intrinsic ab-

[3] *The Vectorial Photoelectric Effect in Thin Films of Alkali Metals*, H. E. Ives, Phys. Rev. *38*, 1209–1218; 1931; *The Photoelectric Effect From Thin Films of Alkali Metal on Silver*, H. E. Ives and H. B. Briggs, Phys. Rev. *38*, 1477–1489; 1931.

sorbing power" of the film. It may, however, profoundly affect the light intensity within the film, and therefore cause the amount of energy absorbed to vary with wave length, plane of polarization, and angle of incidence in a way quite different from that observed with only two media. This is particularly true if the layer is so thin that the results of multiple internal reflection are important. In such cases, we shall have to distinguish between the "bulk absorbing power" of an opaque layer and the "lamellar absorbing power" of a thin layer.

We shall find (§24) that when the film is neither nearly transparent nor nearly opaque, the mathematical formulae for this lamellar absorbing power are so involved that their physical interpretation is rather vague. But when the film is almost completely transparent, they separate naturally into factors which are quite easy to interpret; and of course they degenerate into those appropriate to two media when the film becomes opaque.

§23. *Intrinsic Absorbing Power:* The instantaneous rate at which energy is absorbed per unit time per unit volume is given by the formula

$$Q = \sigma E^2/4\pi.$$

Here E means the *instantaneous value of the physical electric force*, and is, of course, to be identified with the real component of the complex expressions with which the present paper deals. The question which we raise, therefore, is this:

How much heat energy is absorbed by unit volume of a given substance, in unit time, from a plane wave of unit intensity?

To answer this, we must notice that in the most general case, where E is a rotating vector, it has two components at right angles to one another with amplitudes E_0u and E_0u, respectively. (See §7). These components are 90° out of phase with one another: hence they can be written as $E_0u \cos pt$ and $E_0u \sin pt$, respectively. As these components add vectorially, we have

$$Q = \frac{\sigma}{4\pi} E_0^2(u^2 \cos^2 pt + u^2 \sin^2 pt).$$

This is the instantaneous rate of absorption. The average rate (time average) is obviously

$$\overline{Q} = \frac{p}{2\pi} \int_0^{2\pi/p} Q\, dt = \frac{\sigma}{8\pi} E_0^2(u^2 + u^2),$$

since the average value of either $\cos^2 pt$ or $\sin^2 pt$ is 1/2. But, upon recalling the meanings which were attached to the symbols u, u and E_0 in §7, we readily see that $u^2+u^2 = |l^2| + |m^2| + |n^2|$; hence that

$$\overline{Q} = \frac{\sigma}{8\pi}(|E_x^2| + |E_y^2| + |E_z^2|).$$

Finally we may note that, by (8), (10) and footnote 7 (§6),[4]

$$\sigma = 2NK_0p/\mu$$

whence we have

$$\overline{Q} = \frac{NK_0p}{4\pi\mu}(|E_x^2| + |E_y^2| + |E_z^2|). \tag{59}$$

Now, the mean square value of the electric force (the "electric intensity," or simply the "intensity") is

$$\overline{E^2} = \tfrac{1}{2}(|E_x^2| + |E_y^2| + |E_z^2|);$$

and since we have defined the term "intrinsic absorbing power" to be the ratio $\overline{Q}/\overline{E^2}$, we have

$$A_I = \sigma/4\pi = NK_0p/2\pi\mu. \tag{60}$$

It should be said in passing that the result (59) may not be so obvious as it seems. Its truth for plane or elliptically polarized light is self-apparent; but not for hybrid light in which the phase relations among the components E_x, E_y and E_z are rather peculiar.

We must also note that the results contained in this section are absolutely general. We are at liberty to apply them to any medium, provided we use the optical constants appropriate to that medium.

§24. *Case of Two Media: Bulk Absorbing Power:* The study of reflection and refraction in the case of two media has already been carried out in §§13–15, the final results being presented in Tables 1 and 2. To apply these results to our present purposes we need only recall that our upper medium is a dielectric. The lower one may be either conductor or dielectric, but it must be "semi-infinite." Mathematically this means that it must have no other boundary than the plane $z=0$ which separates it from the upper medium. Physically, it implies only that the possibility of light getting back to this plane by internal reflection can be ignored: hence it is satisfied by any opaque body, and also by transparent bodies whose shapes are such as to assure that the refracted beam eventually emerges from other surfaces than the one through which it entered.

We define the "bulk absorbing power" of any such medium by the formula

$$A_B = 1 - |E_2/E|^2; \tag{61}$$

[4] In this and many equations which follow, we replace the velocity of light, which was denoted by c in §11, by its equivalent $p\Lambda_0/2\pi$, in order to avoid confusion with the direction cosine c that appears constantly throughout this part of our work.

that is, one minus its "reflecting power." From Table 1 we readily find this to be

$$A_{B\perp} = 1 - \left|\frac{k - k_1}{k + k_1}\right|^2 = \left|\frac{2k}{k + k_1}\right|^2 Re\frac{k_1}{k}, \qquad (62\perp)$$

when the electric vector is normal to the plane of incidence; and from Table 2

$$A_{B\parallel} = 1 - \left|\frac{\kappa - \kappa_1}{\kappa + \kappa_1}\right|^2 = \left|\frac{2\kappa}{\kappa + \kappa_1}\right|^2 Re\frac{\kappa_1}{\kappa}, \qquad (62\parallel)$$

when the electric vector is in the plane of incidence.[5]

To get a physical insight into the meaning of (61) and (62), we go back to the equations listed in §2, and to the long footnote in §6. We there find that[6]

$$h = 2\pi i\mu/\Lambda_0, \qquad (65)$$

$$q = (2\pi/\Lambda_0)(N + iK_0). \qquad (66)$$

Hence it follows from the definitions of k and κ (Tables 1 and 2) that

$$k = (N + iK_0)c/\mu \qquad (67)$$

and, since $q/g = h/q$,

$$\kappa = -\mu c/(N + iK_0). \qquad (68)$$

We also note that the total electric force $\mathfrak{E}$ *just above the bounding plane* $z=0$ is the vector sum of the forces in the incident and reflected beams. In the $\perp$ case the x and z components are zero, and the y components are [by equations (2)] $Eme^{iq(ax+by+cz)-ipt}$ and $E_2M_2e^{iq(a_2x+b_2y+c_2z)-ipt}$, which, upon putting $z=0$ and taking the other quantities from Table 1, add up to

$$\mathfrak{E}_y = (E + E_2)e^{iqrx-ipt};$$

hence

[5] The third member of each of these equations is obtained from the second by means of the fundamental identity

$$|a|^2 - |b|^2 \equiv |a + b|^2 Re\frac{a - b}{a + b}, \qquad (63)$$

which is satisfied by any complex numbers a and b. *Re* x means the real part of x.

There is another identity of a similar kind which we shall have occasion to use later, and which may best be listed here for reference. It is

$$Re\ (1/a) = |1/a|^2\ Re\ a. \qquad (64)$$

[6] It need hardly be mentioned that these equations are quite general and therefore apply to any medium provided suitable subscripts are attached. That is, in the lower medium we have $q_1 = 2\pi(N_1 + iK_{10})/\Lambda_0$, etc., the "sub 1" being affixed to the N and K_0 which vary from material to material, but not to Λ_0, the "ether wave length" of the light, which does not.

$$\frac{\mathfrak{E}_y}{E} = \frac{2k}{k + k_1}. \tag{69}$$

In the $\|$ case there are two components of the electric vector. Treating them just as the y-component was treated above, we find that

$$\frac{\mathfrak{E}_x}{E} = -\frac{2\kappa_1}{\kappa + \kappa_1}\sqrt{1 - r^2}, \tag{70}$$

$$\frac{\mathfrak{E}_z}{E} = -\frac{2\kappa}{\kappa + \kappa_1} r. \tag{71}$$

Finally, we may note that the tangential components of the electric vector E_1 *just below the bounding surface* (that is, at $z = 0$) are identical with (69) and (70), as they must be by virtue of Boundary Condition II, §14. The normal component of E_1, however, is not (71) but

$$\frac{E_{1z}}{E} = -\frac{2\kappa}{\kappa + \kappa_1}\frac{g}{g_1} r.$$

Hence we have

$$E_{1z}/\mathfrak{E}_z = g/g_1. \tag{72}$$

This coefficient g/g_1 defines the discontinuity between the normal components of electric force on the two sides of the boundary.[7] Though we have derived it from a particular example in which there are only two media and the bounding surface is a plane, it is nevertheless a perfectly general relation, satisfied at all surfaces where two different media meet. Hence we have the rule that:

At the boundary of any two media reflection and refraction of light take place in such a way that the tangential components of the electric vector E, and the normal component of gE, are continuous.

We now return to (62 $\perp$), and note that, by virtue of (69), it reduces to

[7] When the media are both dielectrics, g/g_1 reduces to ϵ/ϵ_1, and (72) becomes the well-known rule that the normal component of displacement is continuous.

If the lower medium is a perfect conductor, g_1 is infinite, and hence $E_{1z} = 0$, as it should be.

In the more general case, in which both conduction and displacement currents exist, neither of these rules is valid. Instead, the general relation is found from the second of equations (1) to be that $\sigma E_z + \epsilon \partial E_z/\partial t$ must be continuous; for the tangential components of H, and therefore the z-component of curl H, are known to be continuous. As we are dealing with plane waves in which the time factor is e^{-ipt}, this rule requires continuity of $(\sigma - \epsilon ip)E_z$; or what amounts to the same thing, of gE_z.

Finally, we may remark that if the two media are of like permeability, so that $\mu = \mu_1$, g/g_1 is identical with q^2/q_1^2. This explains the appearance of the factor q^2/q_1^2 in the paper referred to in Footnote 3.

$$A_{B\perp} = \left|\frac{\mathfrak{E}_y}{E}\right|^2 Re\frac{k_1}{k}; \qquad (73\perp)$$

while (62||) can be shown to be equivalent to[8]

$$A_{B\parallel} = \left(\left|\frac{\mathfrak{E}_x}{E}\right|^2 + \left|\frac{g}{g_1}\right|^2\left|\frac{\mathfrak{E}_z}{E}\right|^2\right) Re\frac{k_1}{k}. \qquad (73\parallel)$$

The interpretation of these equations scarcely needs comment: in each case the first factor is the ratio of the intensity just below the surface to the intensity of the incident beam, while the second factor plays a role somewhat similar to the "intrinsic absorbing power" in equation (59). But it is not the intrinsic absorbing power, nor indeed any *intrinsic* property of the medium, as is evident from the fact that it varies with the angle of incidence.

In fact, the "bulk absorbing powers" (62), or (73), are not truly named at all. They are in reality phenomena of *refraction*, not of *absorption*. Whatever succeeds in passing the bounding surface is absorbed sooner or later, if the lower medium is opaque, or passes out of the system if it is transparent. The amount "absorbed" is therefore determined solely by the reflecting power of the metal; not by its subsequent disposal of the refracted portion.

[8] To establish this, we first note that, by definition,

$$\kappa/\kappa_1 = (c^2k_1/c_1^2k). \qquad (74)$$

Also, it follows from Table 2 that

$$c^2/c_1^2 = c^2 + (r^2q^2c^2/q_1^2c_1^2).$$

We therefore have

$$Re\frac{\kappa}{\kappa_1} = c^2Re\frac{k_1}{k} + r^2Re\left(\frac{q^2c^2}{q_1^2c_1^2}\frac{k_1}{k} + i\epsilon\right), \qquad (75)$$

the term $i\epsilon$ having no effect on the real part of the expression as long as ϵ is a real member. But we may write the last term in the form

$$r^2Re\left[\frac{k_1}{k}\left(\frac{q^2c^2}{q_1^2c^2} + i\epsilon\frac{qc}{q_1c_1}\frac{\mu_1}{\mu}\right)\right], \qquad (76)$$

in which form it is easy to give ϵ such a value that the term in parenthesis will be real. To do this, call $qc/q_1c_1 = u+iv$. Then the parenthesis factors into

$$(u+iv)\left(u+iv+i\epsilon\frac{\mu_1}{\mu}\right)$$

which will obviously be real and equal to $u^2+v^2 = |qc/q_1c_1|^2$ provided we set ϵ equal to $-2v\mu/\mu_1$, as we may do since this value is real. Thus (76) becomes

$$r^2\left|qc/q_1c_1\right|^2Re(k_1/k);$$

and hence by (64) and (75),

$$Re\frac{\kappa_1}{\kappa} = c^2\left|\frac{\kappa_1}{\kappa}\right|^2Re\frac{k_1}{k} + r^2\left|\frac{g}{g_1}\right|^2Re\frac{k_1}{k}.$$

However, as it is this quantity which is spoken of as "absorbing power" in classical optics, it would probably only create confusion to replace it by a more appropriate term.

§25. *Case of Three Media: Lamellar Absorbing Power:* We now turn our attention to the case of three media, separated by the parallel planes $z=0$ and $z=-\zeta$ as shown in Fig. 22, which is, except for minor modifications, a reproduction of Fig. 18.

In our physical application to the problem of photoelectric emission, the middle substance will ordinarily be a thin film of alkali metal, the bottom one the base upon which it is deposited, and the upper one air or vacuum. It will, therefore, save a great deal of circumlocution if we refer

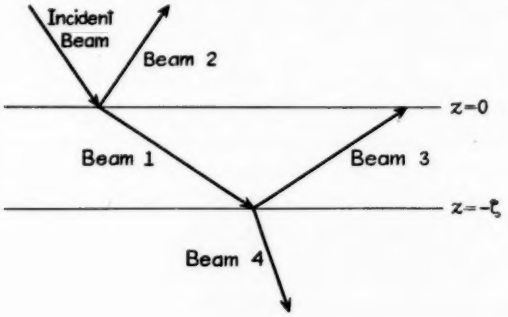

FIG. 22.

to them simply as "film," "base" and "air." However, the results which we shall obtain will be valid no matter what materials are used for film and base, provided only that the upper medium is a transparent dielectric. In other words, unless exceptions are explicitly noted, the only restrictions are that all three materials shall be isotropic and that the extinction coefficient of the upper one shall be zero.

Moreover, so far as the present section is concerned, the film need not be "thin." It can have any thickness from zero (which is a case of two media) to infinity (which is again a case of two media).

A beam of plane-polarized light is incident upon the upper surface of the film. Part of it is reflected as Beam 2, part is transmitted into the base as Beam 4 and the rest is absorbed by the film, in which there are a refracted Beam 1 and an internally reflected Beam 3.

Symbols which refer to these various beams will be supplied with corresponding subscripts. Thus, if they have no subscripts, they refer to the Incident Beam, while if they have subscripts 1, 2, 3 or 4 they refer

TABLE 3. *Reflection and refraction from a film. E normal to the plane of incidence.*

$l=0$	$a=r$	$l'=-c$
$m=1$	$b=0$	$m'=0$
$n=0$	$c=-(1-r^2)^{1/2}$	$n'=a$
$l_1=0$	$a_1=qr/q_1$	$l_1'=-c_1$
$m_1=1$	$b_1=0$	$m_1'=0$
$n_1=0$	$c_1=-(1-q^2r^2/q_1^2)^{1/2}$	$n_1'=a_1$
$l_2=0$	$a_2=r$	$l_2'=-c_2$
$m_2=1$	$b_2=0$	$m_2'=0$
$n_2=0$	$c_2=+(1-r^2)^{1/2}$	$n_2'=a_2$
$l_3=0$	$a_3=qr/q_1$	$l_3'=-c$
$m_3=1$	$b_3=0$	$m_3'=0$
$n_3=0$	$c_3=+(1-q^2r^2/q_1^2)^{1/2}$	$n_3'=a_3$
$l_4=0$	$a_4=qr/q_4$	$l_4'=-c_4$
$m_4=1$	$b_4=0$	$m_4'=0$
$n_4=0$	$c_4=-(1-q^2r^2/q_4^2)^{1/2}$	$n_4'=a_4$

$$
\begin{aligned}
\frac{E_1}{E} &= \frac{2k(k_1+k_4)X^2}{(k+k_1)(k_1+k_4)X^2+(k-k_1)(k_1-k_4)},\\
\frac{E_2}{E} &= \frac{(k-k_1)(k_1+k_4)X^2+(k+k_1)(k_1-k_4)}{(k+k_1)(k_1+k_4)X^2+(k-k_1)(k_1-k_4)},\\
\frac{E_3}{E} &= \frac{2k(k_1-k_4)}{(k+k_1)(k_1+k_4)X^2+(k-k_1)(k_1-k_4)},\\
\frac{E_4'}{E} &= \frac{4kk_1X}{(k+k_1)(k_1+k_4)X^2+(k-k_1)(k_1-k_4)}.
\end{aligned}
\qquad (77\perp)
$$

Notes:

1. The symbol k is used for the combinaton iqc/h, the q, c, and h all being understood to have the same subscripts as the k.

2. The symbol X is used for $e^{iq_1c_1\zeta}$.

3. It is not necessary to write the H's. They are defined by (9), which is valid for any medium, provided q, h, E and H all have the subscript appropriate to that medium.

4. The signs affixed to the radicals denote the signs of the real parts.

to Beams 1, 2, 3 or 4.[9] In addition, since we have no occasion to discuss the values of the various electric and magnetic forces at other places than the bounding surfaces, we shall understand all such symbols to denote values at the upper surface $z=0$ unless a prime is attached, in which case they denote values at $z=-\zeta$.

In order to facilitate reference, we assemble in Tables 3 and 4 the information needed to define the amplitudes, directions of propagation,

[9] Certain quantities which measure intrinsic properties of the three substances necessarily have the same values for the Incident Beam and Beam 2, and also for Beams 1 and 3. The index

TABLE 4. *Reflection and refraction from a film. H normal to the plane of incidence.*

$l=c$	$a=r$	$l'=0$
$m=0$	$b=0$	$m'=1$
$n=-a$	$c=-(1-r^2)^{1/2}$	$n'=0$
$l_1=c_1$	$a_1=qr/q_1$	$l_1'=0$
$m_1=0$	$b_1=0$	$m_1'=1$
$n_1=-a_1$	$c_1=-(1-q^2r^2/q_1^2)^{1/2}$	$n_1'=0$
$l_2=c_2$	$a_2=r$	$l_2'=0$
$m_2=0$	$b_2=0$	$m_2'=1$
$n_2=-a_2$	$c_2=+(1-r^2)^{1/2}$	$n_2'=0$
$l_3=c_3$	$a_3=qr/q_1$	$l_3'=0$
$m_3=0$	$b_3=0$	$m_3'=1$
$n_3=-a_3$	$c_3=+(1-q^2r^2/q_1^2)^{1/2}$	$n_3'=0$
$l_4=c_4$	$a_4=qr/q_4$	$l_4'=0$
$m_4=0$	$b_4=0$	$m_4'=1$
$n_4=-a_4$	$c_4=-(1-q^2r^2/q_4^2)^{1/2}$	$n_4'=0$

$$\frac{H_1}{H}=\frac{2\kappa(\kappa_1+\kappa_4)X^2}{(\kappa+\kappa_1)(\kappa_1+\kappa_4)X^2+(\kappa-\kappa_1)(\kappa_1-\kappa_4)},$$
$$\frac{H_2}{H}=\frac{(\kappa-\kappa_1)(\kappa_1+\kappa_4)X^2+(\kappa+\kappa_1)(\kappa_1-\kappa_4)}{(\kappa+\kappa_1)(\kappa_1+\kappa_4)X^2+(\kappa-\kappa_1)(\kappa_1-\kappa_4)},$$
$$\frac{H_3}{H}=\frac{2\kappa(\kappa_1-\kappa_4)}{(\kappa+\kappa_1)(\kappa_1+\kappa_4)X^2+(\kappa-\kappa_1)(\kappa_1-\kappa_4)},$$
$$\frac{H_4'}{H}=\frac{4\kappa\kappa_1X}{(\kappa+\kappa_1)(\kappa_1+\kappa_4)X^2+(\kappa-\kappa_1)(\kappa_1-\kappa_4)}. \qquad (77\,\|)$$

Notes:

1. The symbol κ is used for the combination iqc/g, the q, c, and g being understood to have the same subscripts as the κ.

2. The symbol X is used for $e^{iq_1c_1\zeta}$.

3. It is not necessary to write the E's. They are defined by (9) which is valid for any medium, provided g, h, E and H all have the subscript appropriate to that medium.

4. The signs affixed to the radicals denote the signs of the real parts.

damping, and states of polarization of the various beams. This material is derived from §§14–17, and its interpretation should be clear in the light of the extended comment of §§2–10. The form of presentation is identical with that used in Tables 1 and 2.

of refraction N, the extinction coefficient K_0 and the propagation constant q belong to this class. In the case of such quantities the subscript 1 is used consistently for the film, even when we are talking about the internally reflected Beam 3, and no subscript at all for air, even when dealing with Beam 2.

With this information before us, we are in a position to compute the z-components of the Poynting vectors (or rather the time-average of these components) for the Beams 1, 2 and 4. As explained in §11, these represent, respectively, the amount of incident energy which falls per second upon a square centimeter of the upper surface, the amount carried away from this square centimeter by the reflected beam, and the amount carried through a square centimeter of the lower surface into the base. If we call these S_z, S_{2z} and S_{4z}, respectively, it is obvious that each square centimeter of illuminated film must absorb $|S_z| - |S_{2z}| - |S_{4z}|$ units of energy per second. The *proportion* absorbed, or the *lamellar absorbing power of the film*, is

$$A_L = 1 - \frac{|S_{2z}| + |S_{4z}|}{|S_z|}.$$

However, since the energy-flow in the incident and transmitted beams is *downward* (that is, in the negative z-direction), while that in the reflected beam is *upward*, the signs of S_z and S_{4z} will be negative, and that of S_{2z} positive. Hence, when algebraic signs are fully accounted for,

$$A_L = 1 + \frac{S_{2z}}{S_z} - \frac{S_{4z}}{S_z}. \tag{78}$$

We now proceed to the evaluation of S_z, S_{2z} and S_{4z}, taking the $\perp$ case first.

As the upper medium is a dielectric, the electric and magnetic vectors are in phase and hence

$$S_z = (p\Lambda_0/16\pi^2)[EH]_z,$$

where the brackets denote the vector product. But, in the $\perp$ case, $[EH]_z = [EH_z] = |EH|c$, and therefore

$$S_z = (p\Lambda_0/16\pi^2)\,|EH|\,c.$$

We may remark in passing that, by the use of (8), (9) and (10), it is possible to throw this into the alternative forms

$$S_z = (p\Lambda_0/16\pi^2)\,|E^2|\,k = -(p\Lambda_0/16\pi^2)\,|H^2|\,\kappa, \tag{79}$$

which we shall have occasion to use later on. It also follows that, if the beam is of unit intensity, (that is, if $|E|^2 = 2$) the amount of energy which falls upon one square centimeter of the plane $z = 0$ is by (67)[10]

[10] The negative sign is explained as follows: Since the beam is directed downward, c, k, and therefore S_z also, are negative. But by "amount of energy" we evidently mean an essentially positive quantity. Hence we must replace the negative numbers c and k by positive numbers $-c$ and $-k$.

$$-p\Lambda_0 k/8\pi^2 = -p\Lambda_0 Nc/8\pi^2\mu.$$

By an exactly similar argument we find the z-component of the Poynting vector in the reflected beam to be

$$S_{2z} = -(p\Lambda_0/16\pi^2)\,|\,E_2{}^2\,|\,k = (p\Lambda_0/16\pi^2)\,|\,H_2{}^2\,|\,\kappa. \tag{80}$$

Dividing this equation by (79), and replacing $|E_2/E|^2$ by its value as given in Table 3, we have[11]

$$\frac{S_{2z}}{S_z} = -\left|\frac{(k-k_1)(k_1+k_4)X^2+(k+k_1)(k_1-k_4)}{(k+k_1)(k_1+k_4)X^2+(k-k_1)(k_1-k_4)}\right|^2. \tag{81 $\perp$}$$

This result has been obtained for the $\perp$ case. It is physically obvious, however, that (79) and (80) must be true for the other case also, since the relationship of the intensity of the light to the magnitude of its electric force-vector does not depend upon the plane of polarization; and this conclusion is easily verified by direct computation of S_z and S_{2z} from the data of Table 4. If, then, we again divide (80) by (79) and replace $|H_2/H|^2$ by its value as given in Table 4, we get the lamellar reflecting power of the film for the $\|$ case. It is

$$\frac{S_{2z}}{S_z} = -\left|\frac{(\kappa-\kappa_1)(\kappa_1+\kappa_4)X^2+(\kappa+\kappa_1)(\kappa_1-\kappa_4)}{(\kappa+\kappa_1)(\kappa_1+\kappa_4)X^2+(\kappa-\kappa_1)(\kappa_1-\kappa_4)}\right|^2. \tag{81 $\|$}$$

We are now ready to turn our attention to the light which passes into the base. Here the situation is a bit more complicated owing to the fact that this beam is "hybrid." Again, however, we may follow the procedure laid down in §11.

We take first the $\perp$ case, and note that, since E_4' lies in the y-direction, we need only use the x-component of the magnetic force in our computation of S_{4z}. However, by Table 3, the x-component of magnetic force is readily found to be $H_4'l_4'$, which, by (9) and Table 3 is easily seen to be given by

$$H_4'l_4'/E_4' = -iq_4c_4/h_4 = -k_4.$$

If the base is a metal, k_4 is a complex quantity.[12] Calling it

$$k_4 = Re^{i\eta},$$

[11] This is *the lamellar reflecting power of the film in the presence of the base.* It reduces to the bulk reflecting power of the base when the thickness of the film is made zero (this corresponds to $X=1$), and to the bulk reflecting power of the film material alone, provided the thickness of the film is made infinite (this corresponds to $X=0$). As we have said, the negative sign merely reflects the fact that the energy flows downward in the incident beam and upward in the reflected one.

[12] This is also true even when the base is a dielectric, if total reflection takes place at the

the argument of §11 shows that

$$\begin{aligned} S_{4z} &= (p\Lambda_0/16\pi^2)\,|\,E_4'H_4'l_4'\,|\cos\eta \\ &= (p\Lambda_0/16\pi^2)\,|\,E_4'\,|^2R\cos\eta \\ &= (p\Lambda_0/16\pi^2)\,|\,E_4'\,|^2\,Re\,k_4. \end{aligned}$$

Dividing this by (79), and taking the value of $|E_4'^2/E^2|$ from Table 3, we get

$$\frac{S_{4z}}{S_z} = \left|\frac{4kk_1X}{(k+k_1)(k_1+k_4)X^2+(k-k_1)(k_1-k_4)}\right|^2 Re\,(k_4/k). \qquad (82\perp)$$

This formula applies to the $\perp$ case. For the $\|$ case we find without great difficulty that

$$\frac{S_{4z}}{S_z} = \left|\frac{4\kappa\kappa_1X}{(\kappa+\kappa_1)(\kappa_1+\kappa_4)X^2+(\kappa-\kappa_1)(\kappa_1-\kappa_4)}\right|^2 Re\,(\kappa_4/\kappa). \qquad (82\,\|)$$

By substituting (81) and (82) in (78) we find the absorbing power of the film in the presence of its backing plate. The formula thus obtained, however, may be considerably simplified by means of certain tedious though not especially difficult algebraic transformations. As they are purely formal, and involve no physical concepts whatever, we shall omit them and state our result at once in the final form which appears to be most convenient for purposes of calculation:

$$\begin{aligned} A_{L\perp} = 2\Bigg[&|\,(k_1+k_4)X+(k_1-k_4)\,|^2\,|\,k(X+1)\,|^2\,Re\frac{k_1(X-1)}{k(X+1)} \\ &+|\,(k_1+k_4)X-(k_1-k_4)\,|^2\,|\,k(X-1)\,|^2\,Re\frac{k_1(X+1)}{k(X-1)}\Bigg] \\ &\div|\,(k+k_1)(k_1+k_4)X^2+(k-k_1)(k_1-k_4)\,|^2. \qquad (83\perp) \end{aligned}$$

The formula for $A_{L\|}$ is identical, except that all k's are replaced by κ's. We shall refer to it as (83 $\|$).

This equation, then, gives the lamellar absorbing power of the film for any angle of incidence, for any thickness of film and (with the k's replaced by κ's) for either plane of polarization of the incident light. It is valid whether the film and base are conductors or dielectrics. It is even possible for the film and base to be of identical material, in which case

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lower boundary of the film. Our argument, though nominally carried out for a metallic base, is good for dielectric bases also, whether totally reflecting or not, since real values of k_4 merely imply that $\eta=0$.

(83) gives the proportion of the incident radiation which is absorbed by a layer of this material adjacent to the bounding surface.

§26. *Approximate Formulae: Lamellar Absorbing Power of Very Thin Films:* We have already indicated that the purposes of this study have to do primarily with very thin films, so thin in fact that they are virtually transparent. For such films we can derive approximate formulae which are both simpler than the ones given in §25, and also easier to interpret physically.

To this end, it is convenient to introduce the notation.

$$\epsilon = iq_1c_1\zeta = k_1h_1\zeta, \tag{84}$$

and assume that this ϵ is a small quantity.[13] Then we have approximately

$$X \doteq 1, \quad X - 1 \doteq \epsilon, \quad X + 1 \doteq 2; \tag{85}$$

and therefore (83 $\perp$) becomes

$$A_{L\perp} \doteq \left|\frac{2k}{k + k_4}\right|^2 \left[Re\frac{k_1\epsilon}{k} + \left|\frac{k_4\epsilon}{k_1}\right|^2 Re\frac{k_1}{k\epsilon}\right]. \tag{86 $\perp$}$$

By an identical argument we also get from (83 $\|$)

$$A_{L\|} \doteq \left|\frac{2\kappa}{\kappa + \kappa_4}\right|^2 \left[Re\frac{\kappa_1\epsilon}{\kappa} + \left|\frac{\kappa_4\epsilon}{\kappa_1}\right|^2 Re\frac{\kappa_1}{\kappa\epsilon}\right]. \tag{86 $\|$}$$

The formal analogy between the $\perp$ and $\|$ cases ends at this point, and we shall therefore find it most convenient to drop the $\|$ case for a time and concentrate upon the study of (86 $\perp$).

To begin with, we must recall that the upper medium is a dielectric. Hence by (67)

$$k = Nc/\mu$$

is real. Therefore, upon using (66), (67) and (84), we find without difficulty that

[13] The physical significance of this assumption is easily seen. As explained in §§4 and 6, the real part of $iq_1c_1\zeta$ measures the amount by which light is attenuated in its travel from the upper to the lower surface of the film. The imaginary part measures the phase change that takes place in the same vertical distance (which is less than the phase change which takes place in the same distance when measured in the direction of propagation). To say that ϵ is small, therefore, means that the film is thin enough to be nearly transparent, and also that it is thin when compared with the wave length of the light in the film (not its ether wave length). In order to meet the latter restriction ζ ought not exceed some such value as $\Lambda/20$.

In the experimental study with which the present paper is mainly concerned, the film is usually a monomolecular layer of an alkali metal. For such films the conditions are probably met for ether wave lengths greater than 100A.

$$Re(k_1/k\epsilon) = 0,$$

$$Re(k_1\epsilon/k) = -4\pi\mu N_1 K_{10}\zeta/c\mu_1\Lambda_0 N.$$

Finally, noting that $-c$ is the cosine of the angle of incidence, (86 $\perp$) takes the form

$$A_{L\perp} \doteq \frac{\left(\frac{N_1 K_{10} p}{2\pi\mu_1}\right)\left(\left|\frac{2k}{k+k_4}\right|^2\right)\zeta}{\left(\frac{N\Lambda_0 p\ \cos I}{8\pi^2\mu}\right)}. \qquad (87\ \perp)$$

For purposes of physical interpretation (87 $\perp$) has been divided into four factors. We see from (60) that the first in the numerator is the intrinsic absorbing power of the material of which the film is composed. That is, it is the rate at which heat energy is produced in unit volume of film material by a wave of unit electric intensity, *this intensity being measured inside the film.*

The interpretation of the second factor is clear from (69). It is the ratio of the total intensity *just above the surface of the film* to the intensity of the incident beam, computed as if the film were absent and the reflection took place directly from the base. It need hardly be noted that by Boundary Condition II, §14, the intensity *just below the surface* is the same as that just above.

The factor in the denominator is the rate at which energy is carried to unit area of film by an incident beam of unit intensity.

The product of these three factors, then, gives the ratio of the energy absorbed per unit volume of film to the amount falling upon unit area of film.

Finally the fourth factor, ζ, is the volume of film material per square centimeter of exposed area.

Turning now to the $\|$ case and applying the transformation (64) to the last term, we readily throw (86 $\|$) into the form

$$A_{L\|} \doteq \left|\frac{2\kappa}{\kappa+\kappa_4}\right|^2 Re\frac{\kappa_1\epsilon}{\kappa} + \left|\frac{2\kappa_4}{\kappa+\kappa_4}\right|^2 Re\frac{\kappa\epsilon}{\kappa_1}. \qquad (88)$$

But by using (84); (65), (66), (67) and (68); and then finally (8), we readily find that

$$Re\frac{\kappa_1\epsilon}{\kappa} = r^2\left|\frac{g}{g_1}\right|^2\left(\frac{N_1 K_{10} p}{2\pi\mu_1}\right)\left(\frac{8\pi^2\mu}{N\Lambda_0 p\ \cos I}\right)\zeta,$$

and

$$Re\frac{\kappa\epsilon}{\kappa_1} = (1 - r^2)\left(\frac{N_1K_{10}p}{2\pi\mu_1}\right)\left(\frac{8\pi^2\mu}{N\Lambda_0 p \cos I}\right)\zeta.$$

Inserting these in (88), and arranging the terms somewhat after the fashion of those in (87 $\perp$), we get finally:

$$A_{L\|} \doteq \frac{\left(\frac{N_1K_{10}p}{2\pi\mu_1}\right)\left(\left|\frac{2\kappa_4}{\kappa+\kappa_4}\right|^2(1-r^2) + \left|\frac{g}{g_1}\right|^2\left|\frac{2\kappa}{\kappa+\kappa_4}\right|^2 r^2\right)\zeta}{\left(\frac{N\Lambda_0 p \cos I}{8\pi^2\mu}\right)}. \qquad (87\,\|)$$

We have again four factors, of which three are identical with three in (87 $\perp$). The meaning of the other is easily seen by reference to (70), (71) and (72). It contains the components of electric intensity *just above the film*, computed as if the film were absent, with the normal component corrected for the discontinuity that must take place on passing the boundary.

The net result of the argument then is this:

In the case of thin[14] *films, the lamellar absorbing power can be found to a high degree of accuracy by*

(a) *ignoring the presence of the film and finding the components of total electric intensity that would exist just above the surface if reflection took place directly between air and BASE* [*by using* (69), (70) *and* (71)];

(b) *correcting the normal component for the discontinuity which must take place in crossing the surface between air and FILM* [*by using* (72)];

(c) *finding the rate at which energy is absorbed by the film material from a wave of this amplitude* [*by use of the intrinsic absorbing power* (60) *and the thickness* ζ];

(d) *and finally dividing this by the rate at which energy is conveyed to the film by the incident beam.*

In other words, both (87 $\perp$) and (87 $\|$) can be written in a single form:

$$A_L \doteq \frac{\left(\frac{N_1K_{10}p}{2\pi\mu_1}\right)\left(\left|\frac{\mathfrak{E}_x}{E}\right|^2 + \left|\frac{\mathfrak{E}_y}{E}\right|^2 + \left|\frac{g}{g_1}\right|^2\left|\frac{\mathfrak{E}_z}{E}\right|^2\right)\zeta}{\left(\frac{N\Lambda_0 p \cos I}{8\pi^2\mu}\right)}, \qquad (89)$$

which applies to *any* plane of polarization; it being understood that the total electric intensities $\mathfrak{E}_x$, $\mathfrak{E}_y$, $\mathfrak{E}_z$ are those which would exist *if reflection took place directly from the base.*

[14] That is, thin enough to justify the use of the approximations (85). (See Footnote 13.)

It may be noted in passing that the only factors of (89) that depend upon the character of the film material are its intrinsic absorbing power and the discontinuity ratio g/g_1, neither of which is in any way dependent upon the angle of incidence of the light, or upon its state of polarization.

§27. *Some Numerical Illustrations:* As a first illustration of the use to which the formulae of §§23–26 may be put, we may compute the bulk absorbing power of rubidium, using the optical constants $N=0.134$,

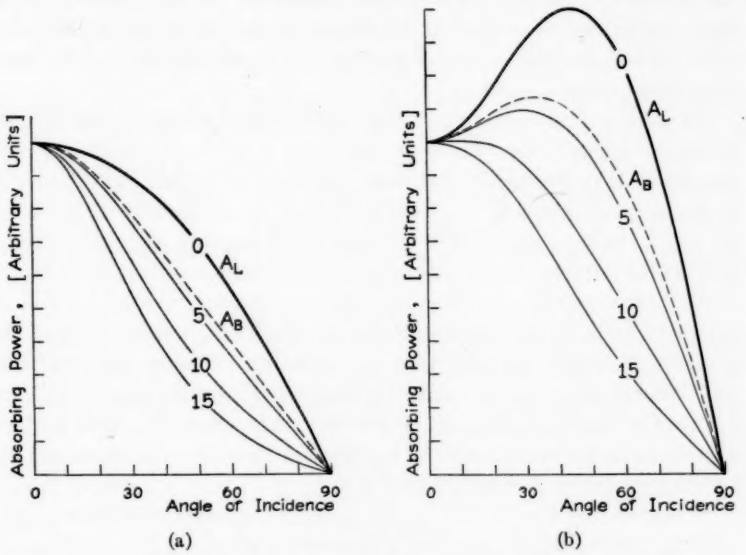

FIG. 23. ABSORBING POWER VS. ANGLE OF INCIDENCE

The electric vector in (a) is normal to, and in (b) parallel to, the plane of incidence.
The optical constants used are those of rubidium at $\Lambda_0=4893$ A: $N=0.134$, $K_0=0.868$.
The dotted curve is bulk absorbing power; the heavy curve lamellar absorbing power of rubidium on rubidium; and the lighter curves the lamellar absorbing powers of thin layers at depths below the surface (in cm$\times 10^{-6}$) indicated by the numbers on the curves.

$K_0=0.868$ corresponding to wave length $\Lambda_0=4893$A. For this purpose, of course, we make use of equations (62 $\perp$) and (62 $\parallel$) of §24.

The result is shown by the dotted curves of Fig. 23 for the two planes of polarization.

As a second example, we may consider the lamellar absorbing power of rubidium on rubidium: that is, the absorbing power of a very thin layer near the surface of a rubidium plate. It is given by equation (89)

of §26, provided we assign the constants of rubidium to both the film and the base.

The only factors in this expression which vary with the angle of incidence are $|\mathfrak{E}_x/E|^2+|\mathfrak{E}_y/E|^2+|g/g_1|^2|\mathfrak{E}_z/E|^2$ and cos I. The other factors are unimportant so long as we are willing to plot our results in arbitrary units. But the factor $|\mathfrak{E}_x/E|^2+|\mathfrak{E}_y/E|^2+|g/g_1|^2|\mathfrak{E}_z/E|^2$ is the electric intensity *just below the surface*, which we have already plotted as the curves marked 0 in Fig. 21. To get the lamellar absorbing power of rubidium on rubidium, therefore, we need only multiply these upper curves of Fig. 21 by the secant of the angle of incidence, thus getting the heavy curves of Fig. 23. The deviation from the bulk absorbing power is striking.

We may go a step further in this connection. We know that the rate at which energy is absorbed by a thin layer *at any depth within the rubidium plate* is proportional to the electric intensity at that depth: that is, to the various curves of Fig. 21. If, then, we divide each of these curves by cos I, (which enters (87) in computing the rate at which energy falls on the surface) we will obtain a set of curves showing what proportion of the incident energy is absorbed at various depths. Plotted in arbitrary units, so as more clearly to show their different behavior as the angle of incidence changes, they are given by the light curves of Fig. 23. Calling these, for the moment, "lamellar absorbing powers at various depths," we may state the rather obvious fact that the curve for bulk absorbing power versus angle of incidence is a sort of average among the lamellar curves at various depths.[15]

In saying that this is obvious, however, I cannot refrain from adding that the dissimilarity of the lamellar curves themselves is a consequence of the fact that light is damped out more rapidly when incident obliquely than when incident normally; and I do not believe *this* fact is either obvious or well known.

Digressing for a moment to the subject of photoemission, we may remark that *if a correlation is to be expected at all between the photo emissivity of bulk material and its optical absorbing power, we would expect it to be found in connection with the dark lamellar curve rather than the dotted bulk curve;* since the electrons emitted at appreciable depths below the surface will probably never emerge.

As a third example, we may consider the variation of absorbing power

[15] If we had kept the scale factors in mind, instead of plotting everything in arbitrary units as we have done in Fig. 23, the bulk curve would be just the integral of the lamellar curves *with respect to depth.*

with wave length. Silver is one of the few metals for which the optical constants have been reliably determined over a broad spectral region, and fortunately also presents an interesting case from our point of view. In Fig. 24 we give the values of N and K_0 as determined by Minor be-

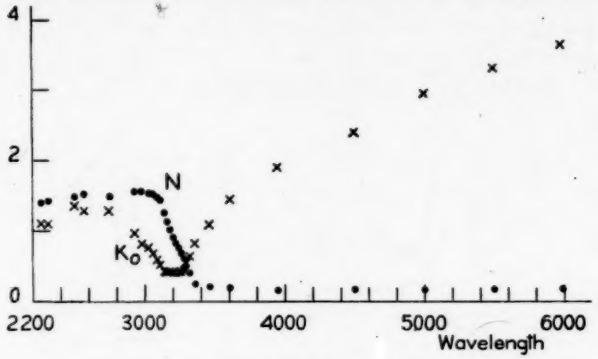

FIG. 24. *Optical constants of silver (Minor's values).*

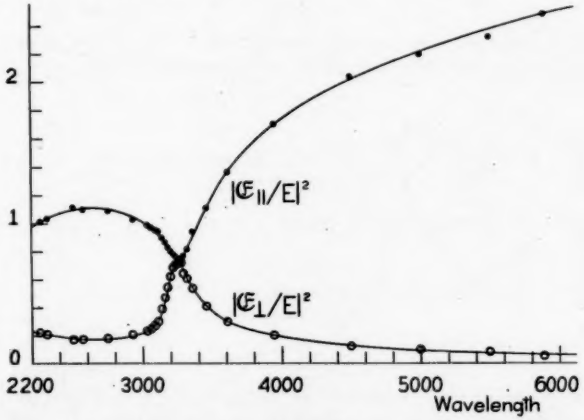

FIG. 25. *Total electric intensity vs. wave length. $\mathfrak{E}$ is the total electric force just above the reflecting surface due to the superposition of incident and reflected beams. Minor's optical constants for silver were used.*

tween $\Lambda_0 = 2263$A and $\Lambda_0 = 5893$A. Using these values in (69), (70) and (71), and confining our attention to a 60° angle of incidence, we readily obtain the curves of Fig. 25 for the electric intensities just above the silver surface (that is, $|\mathfrak{E}_\perp/E|^2 = |\mathfrak{E}_y/E|^2$ for the $\perp$ case, and $|\mathfrak{E}_1/E|^2 = |\mathfrak{E}_x/E|^2 + |\mathfrak{E}_z/E|^2$ for the $\|$ case).

Next we obtain the lamellar absorbing powers as given by (87 $\perp$) and (87 $\|$). We can no longer neglect the denominator and the first factor in the numerator, as we did in computing Fig. 23, since N_1, K_{10} and Λ_0 are all variable. The curves marked $A_{L\perp}$ and $A_{L\|}$ on Fig. 26, therefore,

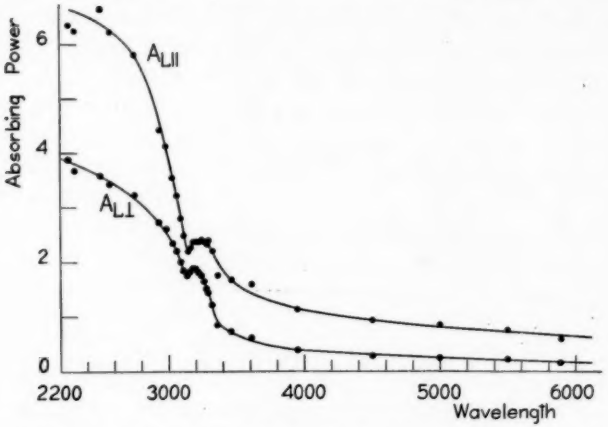

FIG. 26. *Lamellar absorbing power of silver on silver. Computed from Minor's constants. For a film of thickness ζ the vertical scale readings are to be multiplied by $10^5\,\zeta$.*

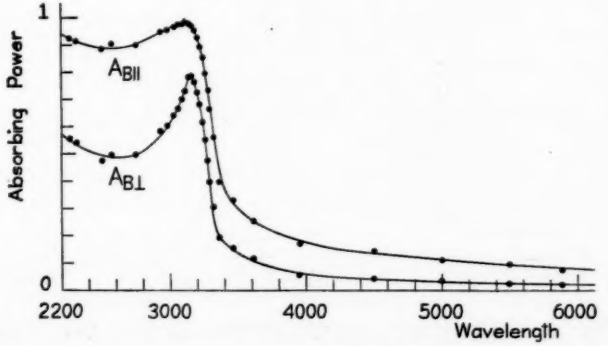

FIG. 27. *Bulk absorbing power of silver. Computed from Minor's constants.*

give quantitatively the proportion of incident light which is absorbed by a thin surface layer of the silver mirror from which it is reflected, so long as the thickness of the layer does not exceed the limits defined in Footnote 13. The scale readings are to be multiplied by $10^5\zeta$.

Finally, we substitute the optical constants of Fig. 24 in (73 $\perp$) and

(73 ||) and obtain the bulk absorbing power of silver over the same spectral region. These are shown in Fig. 27. The great dissimilarity between lamellar and bulk absorption is again apparent.

As a fourth illustration we may take an example in which the film and base are of different substances. For this case we again discuss the variation of absorbing power with angle of incidence, a platinum base with the constants $N_4 = 1.928$, $K_{40} = 3.175$, and a wave length $\Lambda_0 = 4359$A. Ignoring the presence of the film, and using (69), (70) and (71), we com-

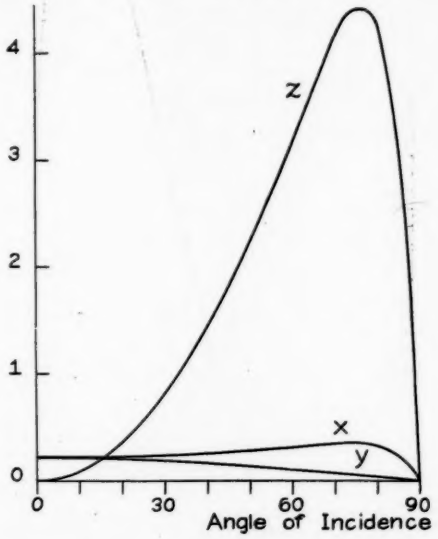

FIG. 28.

The x, y and z components of $|\mathfrak{E}/E|^2$ *sec I vs. angle of incidence.*
The y curve is for $\perp$ *light; the x and z curves for* $\parallel$ *light.*
The optical constants $N = 1.928$, $K_0 = 3.175$ *correspond to platinum at* $\Lambda_0 = 4359$ *A.*

pute the quantities $|\mathfrak{E}_y/E|^2$ sec I, $|\mathfrak{E}_x/E|^2$ sec I and $|\mathfrak{E}_z/E|^2$ sec I, thus getting the results shown in Fig. 28. Except for the factor sec I, these represent electric intensities *just above* the surface, the first one referring to the case of $\perp$ polarization and the other two to the $\parallel$ case. The factor sec I has been introduced at this point purely for convenience: it occurs in every subsequent formula to which we refer, and we save computation by introducing it as early as possible.

In accordance with (87 ||) we may get the lamellar absorbing power $A_{L\parallel}$ of any thin film deposited on this platinum base by multiplying

$|\mathfrak{E}_x/E|^2$ sec I by the $|g/g_1|^2$ appropriate to the *film* material, and adding it to $|\mathfrak{E}_x/E|^2$ sec I. There is in addition, of course, a scale factor com-

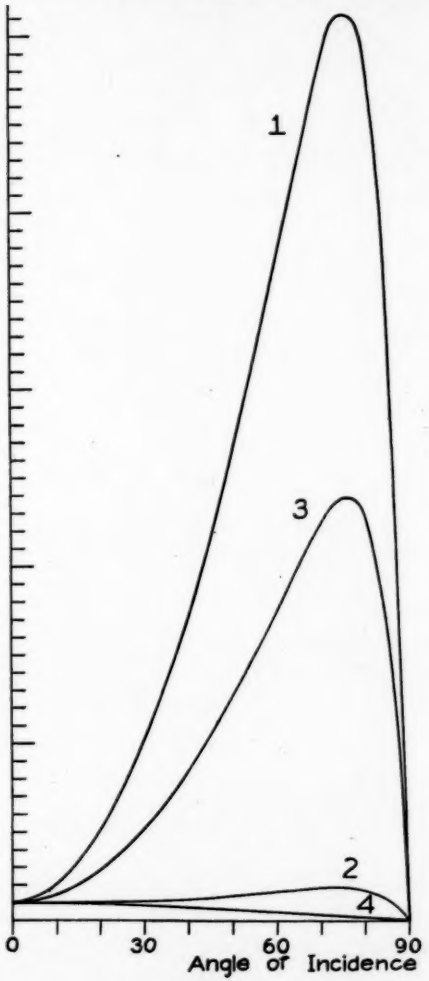

FIG. 29.

(1) *Lamellar absorbing power of potassium on platinum* ($\|$ *case*).
(2) *Lamellar absorbing power of platinum on platinum* ($\|$ *case*).
(3) $|\mathfrak{E}_\parallel/E|^2$ *sec I.*
(4) *Lamellar absorbing power of potassium on platinum, or of platinum on platinum* ($\perp$ *case*); *or* $|\mathfrak{E}_\perp/E|^2$ *sec I.*
The vertical scale is in arbitrary units.

posed of various N's, K_0's and Λ_0's; but this does not vary with the angle of incidence and is of no importance so long as we are interested only in the functional relationship of $A_{L\parallel}$ to I. Similarly, by (87 $\perp$), the lamellar absorbing power $A_{L\perp}$ is (except for a scale factor) independent of the material of the film.

Using these ideas, we plot on Fig. 29 the lamellar absorbing power $A_{L\perp}$ which applies to any substance, and the $A_{L\parallel}$ for films of potassium[16]

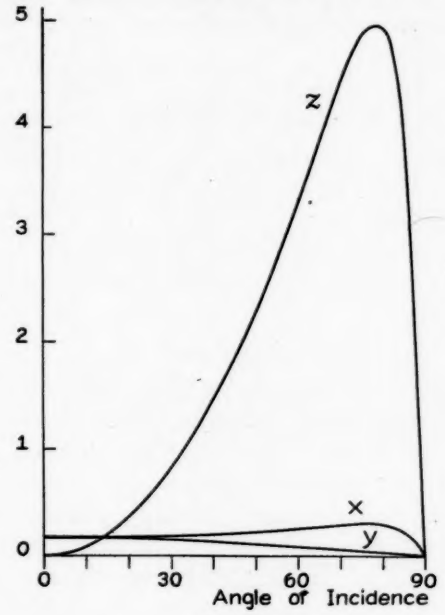

FIG. 30.

The x, y and z components of $|\mathfrak{E}/E|^2$ *sec I vs. angle of incidence.*
The y curve is for $\perp$ *light; the x and z curves for* $\parallel$ *light.*
The optical constants $N=2.438$, $K_0=3.451$ *correspond to platinum at* $\Lambda_0=5461$ *A.*

and of platinum. At this wave length *the ratio of selective to normal lamellar absorption is nearly twice as high for a thin film of potassium on platinum, as for the surface layer of platinum itself.*

For comparison we also plot the curves

$$|\mathfrak{E}_\perp/E|^2 \sec I = |\mathfrak{E}_y/E|^2 \sec I,\ |\mathfrak{E}_\parallel/E|^2 \sec I = (|\mathfrak{E}_x/E|^2 + |\mathfrak{E}_z/E|^2) \sec I$$

which represent the intensity just above the surface multiplied by sec I.

[16] The constants (adopted from Duncan's data) are $N_1=0.0690$; $K_{10}=0.815$.

As an illustration of the great changes that may be produced in lamellar absorption by relatively small changes in the optical constants of the film we include also Figs. 30 and 31, which are identical with Figs. 28 and 29 except that they refer to a wave length $\Lambda_0 = 5461$A. Though the platinum constants $N = 2.438$, $K_0 = 3.451$, are now somewhat different than before, they produce little change in Fig. 30, in the $A_{L\perp}$ curve, (Fig. 31) or in the $A_{L\parallel}$ curve for a platinum film (Fig. 31).

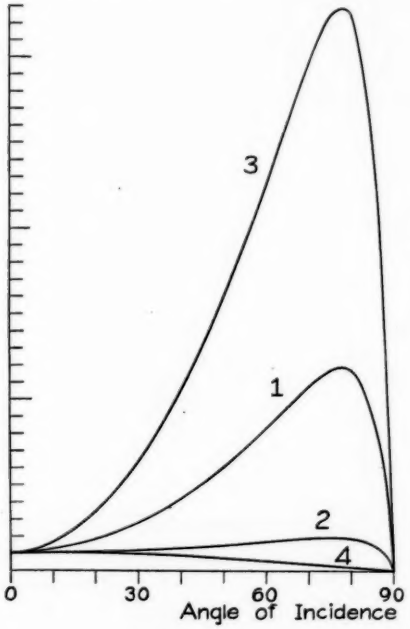

FIG. 31.

(1) *Lamellar absorbing power of potassium on platinum* ($\parallel$ *case*).
(2) *Lamellar absorbing power of platinum on platinum* ($\parallel$ *case*).
(3) $|\mathfrak{E}_\parallel/E|^2$ *sec I*.
(4) *Lamellar absorbing power of potassium on platinum, or of platinum on platinum* ($\perp$ *case*) *or* $|\mathfrak{E}_\perp/E|^2$ *sec I*.
The vertical scale is in arbitrary units.

But the $A_{L\parallel}$ curve for a potassium film rises only half as high as the $\mathfrak{E}_\parallel$ curve in Fig. 31, whereas in Fig. 29 it was twice as high. The constants for potassium, $N_1 = 0.0691$; $K_{10} = 1.325$, were again taken from Duncan's data.

RETINAL CHROMATIC FIELDS AS A FUNCTION OF WAVE LENGTH

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[406 West Oak Street, Lodi, California. Received March 2, 1932]

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I. Introduction

When Thomas Young turned his attention to the subject of color vision, two schools of thought, designed chiefly to explain after-image effects, dominated the field. According to one view, exemplified by Venturi,[1] the changing hues of these phenomena proved the existence of a multiple function for each retinal nerve ending. The other school, however, typified by Scherffer[2] and Bonnet,[3] insisted that this evidence should be explained on a basis of a single function for each retinal particle.

The latter view is represented by the Young-Helmholtz theory, an elaboration of Young's theory of 1801; the former, by the theory which Hering propounded three-quarters of a century later.

Young's theory postulates three elementary stimuli, each corresponding to a single sensitive system of the retina. Helmholtz and König found experimentally sets of three stimuli which, in suitable mixtures,

[1] Dei Colori Imaginarii, Opusc. scelti sulla Scienze, da Carlo Amoretti, Soave V. 21, p. 274.

[2] Direct quotation from work of 1761, in Burch, Roy. Soc. London, Proc., *66*, 204–206; 1899–1900.

[3] Essai Analytique sur l'Âme, et Essai de Psychol., 1785.

N.B. The original works of these three authors were not available to me. The citations are by Burch in reference 2.

would reproduce all the chromatic colors; but they failed to show that the colors corresponding to these stimuli, red, green, and blue or violet, were simple colors.

The most obvious objection to this theory arises from the very strong psychological evidence of a visually simple (or primary) yellow. And since yellow is one of the simple sensations of the four-color theories, these have rapidly gained ground.

The Hering theory, or the chemical and evolutionary adaptation of his theory by Dr. Christine Ladd-Franklin, require two pairs of chromatic processes, values of red and green, and blue and yellow, with each pair belonging to a single visual substance in the retina. Many efforts have been made to demonstrate the existence of these paired processes, partly by establishing the congruence of the retinal areas sensitive to the two members of each pair. The results of these and other researches were not wholly convincing and left color vision in a state which caused Dr. Ladd-Franklin[4] to emphasize strongly the inherent weaknesses and contradictions *inter se* of the existing theories.

The subject of color vision is represented in the majority of general texts of physics and psychology by these theories. But numerous modifications, as well as other independent theories, have been proposed; and some of these are so complex that merely to state them would unduly prolong the paper, without correspondingly adding to its significance.

In 1927 it occured to the writer that further measurements of the retinal areas might be made informative, particularly by the use of homogeneous stimuli of variable wave length.

II. A Preliminary Experiment Using Pigments

The following year a series of perimetric measurements with pigmented targets was begun under the author's supervision by Mr. C. J. Looser, an optometrist, who used as subjects both patients and volunteers.

The perimeter was of a standard make having interchangeable colored targets movable along an arc of 25.5-cm radius designed to be rotated into any meridian. Illumination was secured from a 60-watt "Daylight" lamp, the center of whose filament was at a nearly constant distance of 21 cm from the target. The angle between the plane of the filament and that of the arc was constant. The preëxposure and surrounding field were of neutral gray, roughly judged to be slightly less

[4] Colour and Colour Theories, New York, 1929, p. 114 et passim.

bright than the colored targets. The target discs appeared to be of moderately high saturation, and subtended a visual angle of 1°8′. The fixation point was a dark spot subtending an angle of 40′ against a larger spot of dark ivory. The measurements were made by moving the target slowly inward until the subject reported the appearance of a color.

In this manner a total of 773 eyes were measured with red, green, and blue pigments, along eight meridians, giving 6,184 points for each color.

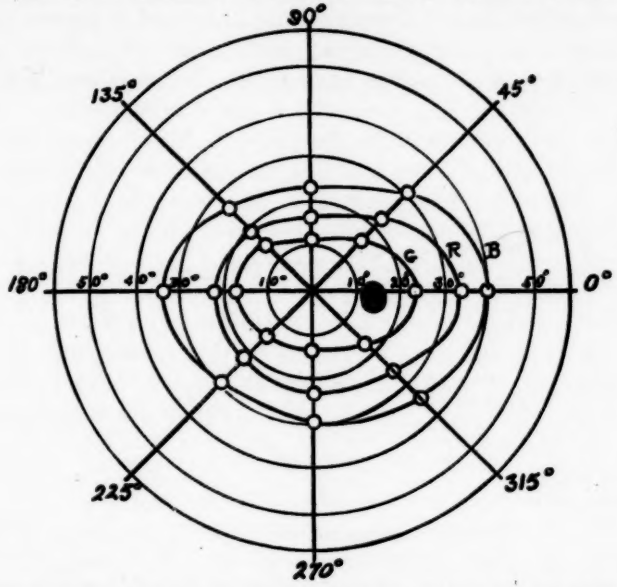

FIG. 1. *Green, red, and blue fields from perimetry with pigment colors. The results of 402 right eyes.*

The average fields for each eye were plotted separately, with the result that no coëxtensive fields were found. The average curves for the right eyes are shown in Fig. 1. It was also noted that careful observers frequently reported the green as yellow in an outer field, and this effect was not entirely eliminated even by the use of decidedly bluish green pigments.

These results are in good agreement with those of Abney,[5] showing that his curves, taken from less extensive data, are none the less characteristic. However, for reasons which appear later in the discussion, the non-coincidence of the red and green sensitive areas could not be taken to prove their structural independence.

[5] Abney, Sir Wm. de W., Researches in Colour Vision, London, 1913, pp. 195–198.

III. THE MEASUREMENTS WITH HOMOGENEOUS LIGHT

THE ARRANGEMENT OF THE APPARATUS

The apparatus was exceedingly simple, as shown in Fig. 2. A framework OQR was designed to rotate about a vertical axis through O, the outer extremities Q and R being carried by heavy rubber-tired rollers with a $1\frac{1}{4}$-inch flat tread. The track consisted of hard-surfaced wallboard tacked to the floor. Surmounting the framework was a table $MNmn$, the whole necessarily being rigid. A separate board KL, pivoted at Z carried the optical system, which consisted of the carbon arc A, condensers B, heat dissipating shield E, of No. 18 B. & S. gauge copper,

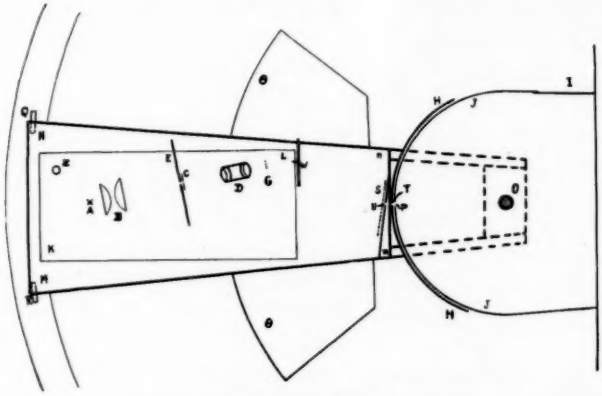

FIG. 2. *The apparatus for perimetry with homogeneous light.*

an adjustable source slit, C, an achromatic collimator D, and the grating G. One of the first order spectra appeared at S; and the portion of this spectrum lying within the slit U illuminated the translucent glass target T, filling a circular opening in shield H. The field thus illuminated subtended 49′ at the eye of the observer. Both U and H were rigidly attached to the table $MNmn$. The wave length was varied by sliding the board KL over the table top about its axis Z by means of a slow motion screw. The position of the spectrum was determined from an arbitrary scale W having 376 usable divisions. Eventually this scale was converted into wave length from spectrometer readings upon the illuminated target from within the enclosure.

In order to restrict the variation of intensity to a relatively narrow range, the current and voltage at the arc were kept at nearly constant values, the power consumption being approximately 2,000 watts, and

two images of the crater were projected as pin-hole images onto screens having crater outlines marked upon them. By thus controlling the power, and the position and characteristics of the crater, it was possible to avoid variations materially affecting the limits of color sensitivity.

A seat for the observer was located upon a stationary platform above the moving framework around *O*, upon the floor of the cylindrical enclosure *J*. A horizontal strip cut from *J* made the target and part of the movable shield *H* visible to the observer. All parts of the enclosure within the observer's field of view were painted a neutral gray. Sufficient light was admitted overhead and behind the observer, from the the general daylight or artificial illumination of the outside room, to make the interior visible after a period of adaptation. The entrance I, being well out of the field of vision, was covered by a black cloth. The fixation point *P* was carried by the stationary enclosure and consisted of a double-ground glass disc, faintly illuminated by a 15-watt daylight lamp after the rays had passed through some gray white tissue paper.

The angle between the target and the fixation point was read from a protractor having circle of 130-cm radius divided to tenths of a degree. Directly above the axis *O* was mounted a triangular sight, which, together with a chin rest, served to locate either eye at the center of the arc described by the target.

THE CONDITIONS OF MEASUREMENT

In making the measurements an observer was seated in the enclosure, and after the lapse of several minutes to permit time for adaptation, he fixated the point *P*. The surroundings were then visible as a very dark gray, the movable shield being scarcely distinguishable from the remainder of the neutral gray interior of the enclosure. The target was now illuminated at the desired wave length and was brought slowly toward the center of the field of view by rotating the table and framework structure about *O*. The subject called the first appearance of any chromatic color, whereupon the operator read the angle from the large protractor. Since the recording was done by an assistant, it was not necessary to stop the motion to get the reading; thus other sensations might be noted as they appeared in the course of the continued movement. Every measurement consisted of at least three independent readings in close agreement.

None of the equipment outside the enclosure was visible to the observer, and there were no marks by which he could estimate duplicate positions of the target. Stray light from the outside room was excluded

to the extent that, with the chromatic illumination cut off, the target showed no contrast against the background. A ventilating fan which ran at all times ordinarily prevented the observer from hearing the readings. The subjects were not instructed as to what color to expect, although they were sometimes permitted to look directly at the illuminated target with both eyes before the observations began. In the latter case they were warned that the color first observed might or might not be that which they had just seen. The prejudicial effect of the preview of the stimulus was found to be neglible.

The observers were in no case familiar with any theory of color vision. The best were art students who were extremely sensitive to color variations, but satisfactory results were also obtained from others chosen at random from medicine, engineering, mathematics, chemistry, english, music, biology, etc.

It will already have been noticed that the design of the apparatus restricts the measurements to the horizontal meridian. Most of the measurements were taken on both the nasal and temporal sides of the retina and were of the same general nature, with the expected greater extension of the former. But the nasal values were not made use of in the data because of the frequent interruption by the blind spot. The limits, field sizes, sensitive areas, etc., referred to in the following discussion will, therefore, be merely the horizontal extension on the temporal side of the retina.

THE MEASUREMENT AT SELECTED WAVE LENGTHS

As a starting point, measurements were begun with an arbitrary wave length for each of four colors, red, green, yellow, and blue. The wave lengths were such as to produce these colors for the majority of observers and were 638, 528, 593, and 464 mμ, respectively. The limits for these four colors were paired for each eye in six combinations and the differences obtained. The average differences, taken regardless of sign, are shown in Table I. It is to be seen that the starred pairs fail to show closer size relationship than the other, supposedly unrelated, pairs.

DISCUSSION OF THESE PARTIAL RESULTS

There are, however, several reasons for not concluding from these data that the sensations are unpaired. The first lies in the choice of stimuli. The mere choice of these stimuli by the observers implies nothing as to the simplicity or complexity of the resulting colors. Experimental evidence that neither the stimulus for red nor the stimulus for green produced these colors alone appeared in the observation of a yellow-sensitive area outside the limits for both the red and the green.

Secondly, the brightness was not the same for all wave lengths, for the distribution of energy in the arc spectrum and the possible selective absorption by the translucent glass have not been taken into account. Ferree and Rand[6] have shown quantitatively the dependence of the sensation limits upon stimulus intensity, although the variation had previously been noted by Aubert,[7] Landolt,[8] and Charpentier.[9]

TABLE I. *Comparison of the field sizes for the arbitrary stimuli.*

Color pair	Average difference in field sizes (degrees)	No. of pairs measured
*Red-green	3.2	70
Red-blue	4.2	67
Red-yellow	4.2	49
Green-yellow	5.7	51
Green-blue	5.3	68
*Yellow-blue	4.3	41

The saturation is a third factor affecting the sensitivity limit.[10] Since the amount of stray light falling upon the target was negligible, the saturation is that for homogeneous stimuli of moderately low intensity.

The effects of the low brightness of the preëxposure and the surrounding field were probably the factors contributing most heavily to the shrinking of the fields,[11] both being considerably darker than the colors themselves.

Three time elements appear to enter into the result. The most apparent effect is a decrease in the fields after repeated observations. Usually, at least six observations could be made with the same color before noticeable shrinking began. Excessive length of time of fixations, regardless of color changes and brief rest periods, often caused erratic behavior and loss of fixation, together with shrinking of the fields. Lastly, the speed of the image in traversing the retina was found, when greatly increased, to produce irregularity; but slight changes of speed,

[6] Ferree, C. E., and Rand, G., *Chromatic Thresholds of Sensation from the Center to the Periphery of the Retina*, Part I, Psychol. Rev., *26*, 16–41; 1919.

[7] Aubert, *Physiologie der Netzhaut*, 1865, p. 116.

[8] Landolt und Snellen, *Handbuch der ges. Augenheilkunde von Graefe und Saemische*, 1874, 3, p. 70.

[9] Charpentier, Augustin, *La Lumière et les Couleurs*, Paris, 1888.

[10] Bull, Ole, *Studien über Lichtsinn und Farbensinn*. Albrecht von Graefe's Archiv f. Ophth., 27, 1881, pp. 54–154.

[11] Ferree and Rand, *The Limits of Color Sensitivity: Effects of Brightness of Pre-exposure and Surrounding Field*, Psychol. Rev. *27*, 377–398; 1920.

such as might occur during the measurements, were observed to have no appreciable effect.

Of course, all of the above discussion can be applied to the preliminary experiment, Section II, with added difficulties arising from the heterogeneous reflecting power of the pigments.

The uncontrolled variables mentioned above have been taken as preventing a conclusion that coëxtensive fields for the supposedly paired sensations are non-existent; but comparisons with some of the previous work might be profitable. Ferree and Rand have done the most extensive work, and the writer's results agree qualitatively with their curves[12] in the region of 1.5×10^{-9} watts, with the exception that the fields are all less extensive. The constriction may be attributed chiefly to the low brightness of preëxposure and surrounding field, the relatively small size of the target,[13] subtending only 49′, compared to nearly $3\frac{1}{2}$ degrees for the authors cited, and also to individual variation.

Charpentier[14] got a result quite contrary to those obtained by Ferree and Rand and by the author for red, green, and blue. He found that for most intensities the limits are, in order from the center, red, green, and blue, with red and green limits much nearer coincident than the limits for the other two possible pairs. But this is contrary to the preponderance of evidence, and moreover, his procedure introduced simultaneously both the inverse variation of intensity with the square of the distance and the effect of decrease in subtended area. Thus his results are not comparable to those under discussion.

Unknown to the writer until recently, the more thorough investigations of field limits by Ferree and Rand[15] had led them not only to conclude that generally coëxtensive fields for red and green were non-existent for stimuli of the same order of intensity, but also to question seriously much of the previous work. They insist that the results of Bull,[16] Hess,[17] and Baird,[18] all of whom claimed coëxtensive limits,[19] show

[12] Op. cit., Psychol. Rev., *26*, 31; 1919.

[13] Abney, Op. cit., pp. 208–210; reading, I presume, "Powers of –2" as "Powers of $\frac{1}{2}$."

[14] Charpentier, op. cit., Curve, p. 202.

[15] Op. cit., Part II, Psychol. Rev., *26*, 150–163; 1919.

[16] Bull, Ole, Op. cit., Especially Figure 2 at the end of Abtheilung 1; also *Sur la périmétrie au moyen de pigments colorés*, Annales d'oculistique, 110, 1893, pp. 169–181, values p. 177.

[17] Hess, Carl, *Ueber den Farbensinn bei indirectem Sehen*, Albrecht v. Graefe's Arch. f. Ophth., *35*, 4, 1889, pp. 1–62, tabular values pp. 45–51.

[18] Baird, J. W., *The Color Sensitivity of the Peripheral Retina*, Carnegie Institution, Washington, 1905, pp. 61–62.

[19] Also cf. Hegg, Emil, *Zur Farbenperimetrie*, Albrecht v. Graefe's Arch. f. Ophth., 1892, 38, 3, pp. 145–168, tabular values p. 166, and *La périmétrie des couleurs*, Ann. d'oculistique, 109, 1893, pp. 321–347, tabular values pp. 343–344.

deviations between the limits of red and green of 1°–3° and for blue and yellow of 1°–5°.

Kirschmann[20] had previously found the deviations more significant than the coincidences, although his failure to observe the effect of either brightness or saturation robbed his results of real significance.

Ferree and Rand have taken considerable pains to show from the standpoint of the change in threshold sensitivity necessary to produce these differences in limits that the deviations are far from negligible. As to the experimental accuracy, the results of the present work show a value of 0.3 degree for the probable deviation of a single reading from the mean of from three to five, this from a total of 2921 individual readings.

The discussion makes it clear that neither the coëxtensive nor the non-coëxtensive limits can be considered as an important factor in the choice of a color theory. However, there is one experimental observation of Ferree and Rand which appears to be highly significant. This is their failure to extend the green at high intensity, even though the red was easily extended sufficiently to interlace with the limits for yellow, blue, and white. There seems to be no question as to the adequacy of their intensity, for they report having passed beyond the intensity giving the maximum area of chromatic sensation. They used both spectral[21] light of 522 mμ and pigments[22] as their stimuli for green. The correctness of their choice of wave length will be discussed in Section IV, with a view to determining whether their stimulus was actually suited to giving the greatest extension of the green. If this point can be satisfactorily cleared up, they have shown that there exists a retinal area where the red process is found unaccompanied by the green. This fact, if confirmed, would seem to the author to be too important and too direct in its implications to be ignored in the color theories.

IV. THE VARIATION OF THE LIMITS WITH WAVE LENGTH

THE CONDITIONS OF MEASUREMENT

The apparatus and general conditions remained as described in Section III, none of the essential adjustments having been altered. However, slight variations in procedure were introduced. At first the points were plotted as previously, from the average of a number of successive meas-

[20] Kirschmann, A., Philosophische Studien von Wundt, *8*, 592–614; 1893.

[21] Op. cit., Psychol. Rev., *26*, 18; 25–37; 1919. *The Absolute Limits of Color Sensitivity*, Psychol. Rev., *27*, 1–23, 21; 1920 (1).

[22] Op. cit., Psychol. Rev., *27*, 18; 1920.

urements of alternate eyes with stimulus at the same wave length, and satisfactory curves were obtained in this manner. But because the time required was so great that only one of the observers was able to withstand the fatigue, it was found desirable to measure only one eye at a sitting, taking a single reading at each wave length. In any case, the whole procedure was repeated at subsequent sittings in order to relocate all the points. At the several sittings, the measurements were begun with different wave lengths and progressed in either direction, thus assuring that the results were not greatly influenced by the order of observation. The points relocated were in satisfactory agreement with the previous values except under the following circumstances: 1. The observer had suffered a marked change in physical condition, as from illness. 2. The observer had been unduly fatigued at one of the sittings, as from chronic loss of sleep. 3. The observer had made radical changes in habits, such as smoking or drinking. In view of the clinical uses of perimetric variations, these results were not unexpected, and they were therefore usually recognized and eliminated.

For wave lengths at which more than one sensation limit appeared, all the successive limits were observed during a single uninterrupted passage of the target from the periphery to the center. Thus none of the slower physiological variations would affect the relations between the various limits at the same wave length.

DISCUSSION OF THE RESULTS AND COMPARISON WITH SOME OF THE PREVIOUS WORK

1. WAVE LENGTHS GIVING THE MAXIMAL AREA; WAVE-LENGTH RANGES OF SENSITIVITY

Fig. 3 summarizes the results. The ordinates are the limits of extent for the several apparent sensations (horizontal meridian, temporal side of the retina), and the abscissae are wave lengths in millimicrons. If differences of intensity in immediately adjacent portions of the spectrum may be neglected, the maximum in each case indicates the wave length producing the most extensive fields. To show that this may be done it is necessary to consider the effects of absorption treated in Section V. There it appears that any considerable decrease in the intensity of the stimulus produces a corresponding decrease in area. Hence the decrease in area on either side of the maxima might be the result of passing into a less intense portion of the spectrum. But if this were true, an effect of sufficient magnitude to be important in the present measurements would be shown by a dipping of any other curve taken at the

same wave lengths. Since the wave lengths through which each inner curve falls off (the upper and lower visual limits excepted) are productive of a second curve which contains no trough or depression indicative of a wave length band of significantly lower intensity, it may be inferred that the major characteristics of the curves are the result of wavelength variation and that the effects of intensity variations from one part of the spectrum to another are of a minor nature. The present crude measurments suffice only for treatment of the grosser effects.

Ferree and Rand[23] used the limits as a measure of the threshold sensitivity of the receptor mechanism (or "sensing fibers"). In this sense, subject to correction for irregular energy distribution in the spectrum, the wave length of the maximum may be said to be that for which a

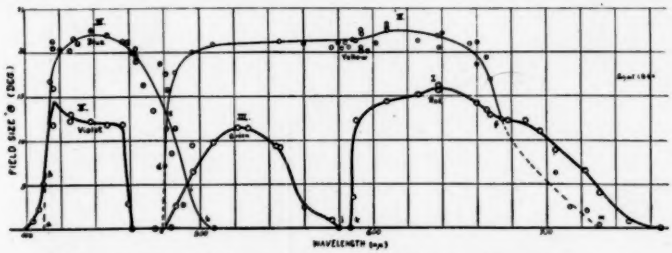

FIG. 3. *The variation of the chromatic field limits with wave length.*

receptor mechanism requires the lowest threshold stimulation in order that sensation should be produced.

The term "receptor" or "receptor mechanism" is used with the intention of avoiding the implication of any theory. Whatever may be the nature of the simple colors, the term is meant to refer to retinal parts or processes which give rise to a component of chromatic color when properly affected by light, regardless of whether the receptor be thought of as unique for each simple color or for each pair of colors.

The points at which the curves reach the axis give the wave lengths at which the stimulus intensity used fell just short of producing the color at any point during the passage of the image from the periphery to the center of the retina; and this of course meant that the chromatic color failed to appear by direct monocular vision as well. These wavelengths, therefore, mark the limits of the wavelength range within which the stimulus, at its present intensity and purity, must lie in order to give rise to the specific chromatic color.

[23] Op. cit., Psychol. Rev., *26*, 150–163; 1919.

The writer is convinced, however, that these ranges and wave-length limits should by no means be considered as fixed. It was pointed out in the course of the discussion of Section III that a large number of investigators have found that, other conditions remaining constant, the retinal fields at any wave length are a function of the stimulus intensity. The curves of Ferree and Rand were cited to show that, at the low intensities used in the present work, the increase of field size with intensity was moderately rapid. However, their curves also show that for intensities low enough to restrict all the chromatic limits to less than about 10° the increase with intensity is so rapid[24] as to seem little short of unstable, the most rapid variation accompanying the intensities barely producing a chromatic color in the central retina.

Since the fields when small are so exceedingly responsive to small increases in intensity, a very slight increase in the effective intensity of excitation suffices to change them from zero to an appreciable size. This leads to the expectation that the base of a curve will be extended to include a wider range of wave length with increase in intensity more rapidly than its maximal limit will be extended toward the periphery of the retina.

One is thus led to doubt whether experimenters who used high intensities, Burch, Ferree and Rand, et al., were able to avoid too-effective stimulation of sensations other than those desired.

This is consistent also with an effect observed by many workers and made use of by Ferree and Rand in their determinations of "Absolute limits", viz., that a greatly intensified chromatic stimulus takes on the appearance of white; for it is merely necessary to assume that the spread and field extension continues until the curves greatly overlap, so that two or more sensations are stimulated sufficiently to produce a cancelling or complementary effect. This would be more satisfactorily explained by assuming that each sensation approaches a limit beyond which increasing stimulation produces little increase in the sensation, provided the limits are of the proper values to give cancelling. It is not intended to exclude other possible explanations, such as increased achromatic activity of the rods to the extent of masking the remaining chroma, etc.

Burch[25] attempted to determine these ranges by completely fatiguing the entire retinal area with portions of a very bright prismatic solar

[24] Op. cit., Psychol. Rev., *26*, 30–31; 1919.

[25] Burch, Geo. J., *On Artificial Temporary Colour-blindness*. Roy. Soc. Lond., Phil. Trans., *191b.*, 1–34; 1898.

TABLE II. *Ranges of sensitivity* ($m\mu$).

	Burch	The Writer	
	Range	Max. extent	Range*
Red	551–760.6	640	585–760
Yellow	Unobserved	610?	480–730 (gross) 580–585 (pure)
Green	484–593	522	480–580
Blue	443–517	440	410–510
Violet	397.5–454	420?	399–460

* Not supposed by the writer to be constant, but to increase with effective stimulus intensity. See previous discussion.

spectrum. It will be seen by the comparison in Table II that the writer's results except for yellow, are in fair agreement with those of Burch, due allowance being made for his much brighter stimulus. His failure to observe the yellow may be explained otherwise than by its absence; e.g., the extremely wide wave-length range of sensitivity for the yellow, together with high intensity of the stimulus, may have caused all his stimuli with the exception of violet, to fatigue the yellow sensation. Moreover, under these conditions, the after images, both positive and negative, might readily pass unnoticed. Thus, for example, during fatigue for blue, if accomplished by means of a stimulus lying also within the range of the yellow, the latter sensation might be affected sufficiently to give, let us say, a positive after image. The combined effect of this with the blue would be a complementary white, which would probably be noted as nothing more than a retardation in the development of the blue. Meanwhile the development of the negative after image for yellow would be observed merely as a prolongation or strengthening of the positive after image for blue; and the effect of the arrival of the negative after image for blue at this stage might be observed only as the fading of the previous effect into white. Moreover, upon fatigue for either red or green, the effect of yellow would appear as a transitory blend during the development of their after images; and the proximity of the wavelength limits for red and green in the central retina, even at low intensity, (Fig. 3, *j* and *k*) leads to the expectation that with more intense stimuli they would overlap in the manner which Burch reported,[26] thus effectively preventing his observation of the yellow alone. McDougall appears to have observed the yellow,[27] although he interpreted the

[26] Op. cit., p. 21.

[27] *v.* Parsons, J. H., *Introduction to Colour Vision*, Cambridge, 1915.

result from the standpoint of a mixed sensation; and he also observed, in agreement with Burch, a longer time interval for the effects of blue. Although both of these facts appear to lend weight to the above view, the explanation has been introduced merely as one of several possibilities which lead the writer to consider that Burch's failure to observe the yellow is by no means proof either of its non-existence or of unreliability in his observations. Furthermore, the immediate purpose is to establish nothing more than the comparableness of his results with those of the present paper, without entering upon a general discussion of the subject of after-image effects.

The wave lengths supposed to correspond to unitary hues, *Urfarben*, have been sought by numerous other experimenters, but none of their methods were as simple and direct as that of Burch, who appears to have had a notion of the general overlapping of the ranges of wave-length sensitivity very similar to that growing out of the present investigation. The relation of the results to those obtained by workers who attempted to produce the unitary hues by means of heterogeneous stimuli must be deferred until the latter part of the discussion.

It has already been mentioned that Ferree and Rand found that, although upon very intense stimulation, the red limits were extended nearly to the limits for white, it was impossible to similarly extend the green, however great the intensity. A perusal of their papers made it appear to the writer that their choice of wave length might be at fault. But their arbitrary wave length for green, 522 mμ,[28] is identical, to the nearest mμ, with the value for the maximum of Curve III in Fig. 3. This agreement, although probably little more than coincidence, should allay any doubt as to the appropriateness of their stimulus. It should be mentioned that the curves of Fig. 3 were plotted several months before the writer first became acquainted with the contents of the Ferree and Rand papers.

2. THE RANGE OF SIMULTANEOUS BLUE AND YELLOW

The explanation of the effects in the range 480–490 mμ involves the heart of the entire discussion. The ordinates give the angle at which, under the conditions of this experiment, the chromatic sensitivity acquires a threshold value low enough for excitation by the stimulus used. Now, the effect observed upon crossing color-field limits lying within more extensive limits is not that of the pure inner sensation, but that

[28] Op. cit., Part I, p. 18.

of the inner compounded with the outer. For example, with 595 mμ yellow appears at about 22°, and reddish-yellow (orange) at about 13°.

Thus, with a wave length of 485 mμ, yellow was observed at 18°; but at 11°, on *ci*, a change was observed to a faintly yellowish gray, which became bluish gray toward the center. The effect was at first thought to be anomalous, but was soon seen to form a continuous branch of the curve for the blue limits, and was thenceforth interpreted as a part of that curve, with blue now superimposed upon the yellow. At 2° green appeared, the observed color being bluish green.

With 480 mμ, blue appeared at about 14° and the yellowish gray at about 7.5°, at *d*, the green appearing as often as not at a few tenths of a degree from the center.

Complex effects were also observed at other wave lengths; the only ranges in which the chromatic color remained unchanged being in the blue from 460 to nearly 480 mμ, in the narrow range *jk* of the yellow (and even here there was an increase in the apparent saturation of the color nearer the center) and, doubtfully, in the extreme red and violet.

3. THE STABLE OR "PHYSIOLOGICALLY PURE" SENSATIONS

According to the usual view, there exist four stimuli whose hue is independent of the part of the retina stimulated, so long as it is within the area giving chromatic vision. The researches leading to this conclusion were based upon the Hering theory and were concurrent with the investigations which gave the coincident fields discussed in Section III.

Bull[29] appears to have been the first to produce these stimuli. His criterion was that in passing from the central retina to the periphery there should be no change in hue other than the change to the "gray" of achromatic indirect vision. His values were blue, yellow, a bluish red, and a bluish green. These were the same as the values with which he found the coincidence of limits already mentioned.

Hess obtained a similar result for his four unveränderlichen Farbentöne, except that he specified chrome yellow, and ultra-marine blue. More important, however, he investigated the spectrum[30] and found three wave-length regions whose color changed from chromatic to achromatic as the peripheral retina was approached, without an intermediate change of hue. The wave lengths were

[29] Loc. cit.

[30] Op. cit., Albrecht v. Graefe's Arch. f. Ophth., *35*, 4, 17–25; 1889.

for Yellow 577 mμ
for Green 500–497 mμ
for Blue 477–475 mμ.

But he was unable to find any spectral red whose hue did not change. He found it necessary therefore to resort to a bluish red which appeared to lie between spectral red and violet. Furthermore, he showed that the pairs red and green, and blue and yellow thus determined were complementary.

Hess[31] obtained somewhat similar results using a Hering color mixer.

Baird verified Hess's conclusion that blue must be added to spectral red in order to produce the invariable sensation; and he found[32] a mean value of 570 mμ for the yellow, using a filter with a range of transmission 551–587 mμ. But for the green he used a filter transmitting the range 483–500 mμ, mean value 490 mμ, and for the blue 448–474 mμ, mean value 460 mμ.

Dreher[33] more recently confirmed the work of Hess.

Let us compare these results with the implications of Fig. 3. The yellow of Hess, under the conditions of the figure, would fall just within the upper wave-length limit of Curve III, and would very definitely give green in the central region. The central stimulation of the green is even more to be expected from Baird's value of 570 mμ.

The invariable greens of both Hess and Baird fall within the overlapping ranges of Curves II and IV. Assuming that the sensitivity ratio for blue and yellow is fairly constant, the wave length corresponding to *c* would be expected to give complementary gray until it reached the limit for green, and the observed effect would be the invariable green. Of course the work of Ferree and Rand already referred to shows that this is not true; but it is still possible for the observed result to have been obtained, since the widths of the spectral bands used by both men might readily have obscured the variation occuring in the relatively short distance from *c* to the center of the retina.

Hess's blue lies in the center of the range covered by Curve IV alone, which, of course means that the writer also found the hue there invariable. Baird's blue here would also lie in the range of Curve V, thus giving a hue variable in two steps.

The necessity of adding blue to the red in order to produce a stable hue is evident from the positions of Curves I and II; the function of the

[31] Loc. cit.
[32] Op. cit., pp. 60, 72.
[33] ZS. f. Psychol. und Physiol. der Sinnesorgane, 46: *2*, 2–82; 1911–12.

blue being merely that of cancelling yellow when the red limit for the particular stimulus value used is reached. Again, the work referred to above shows that this cancelling is not constant and exact, but merely approximate.

On this basis, the explanation of the invariable hues for red and green is not that the stimuli produce simple colors alone, but that, on the contary, they each excite three chromatic components, two of which cancel each other.

4. THE DOUBTFUL PARTS OF THE CURVES

The dotted portions of the curves are those regarded as doubtful. The segment *ab* would be determined by observing a shift toward a bluer cast within the violet. This was apparently done in the case of two observers, but no certainty can be attached to it for the reason that, as called to my attention by Dr. Deane B. Judd, the presence of appreciable stray light in this region would produce a shift toward the red. Helmholtz[34] observed that an increase in the red component of the violet sensation resulted from the addition of white. Abney[35] investigated these effects and stated in his first paper that dilution of blue (4776A) with white light caused the hue to become redder, but that in the violet "the addition of white caused the color to become bluer." However, in his later work he substituted "redder" for "bluer" in the quoted sentence. Helmholtz's result was substantiated by Révész,[36] Schrödinger[37] and Edridge-Green.[38] More recently, Judd in extensive investigations, the full account of which has not yet been published, found that "for wave lengths below 460 mμ a reduction in the purity of the mixture makes the field appear more violet than before."[39] The hue may now be restored to its original value by increasing the dominant wave length of the mixture—i.e., shifting it toward the blue.

However, considerable care was taken to prevent stray light from

[34] v. Helmholtz, H. *Physiologischen Optik*, 2nd Ed., Leipzig, 1896, p. 471.

[35] Abney, Sir. Wm. de W., *On Changes in the Hue of Spectrum Colours upon Dilution with White Light*. Roy. Soc. Lond., Proc., *83A*, 1909, pp. 120–127; and *Researches in Colour Vision and the Trichromatic Theory*, London, 1913, p. 257.

[36] Révész, Géza, *Über die vom Weiss ausgehende Schwächung der Wirksamkeit farbiger Lichtreize*, ZS. f. Psychol. u. Physiol. der Sinnesorgane, 41:2, pp. 116–118; 1907.

[37] Schrödinger, E., *Grundlinien einer Theorie der Farbenmetrik im Tagessehen*, Ann. d. Physik, (4) *63*, pp. 512–517; 1920.

[38] Edridge-Green, F. W., *The Change in Hue Caused by the Addition of White Light to Spectral Colours*, Report of Brit. Assn. for the Advancement of Science, 89th meeting, Edinburgh, 1921, p. 447.

[39] Judd, Deane B., *The Empiric Relation between Dominant Wave length and Purity*, J.O.S.A., *14*, 475; 1927.

falling upon the target, and this, together with the fact that the curves became sensibly zero, both slightly within the ultraviolet limit of the spectrum used and also in the region of high absorption by the didymium glass filter, leads the writer to conclude that the effect of stray light was not appreciable.

Burch[40] reports having been able to fatigue the sensation of violet without fatiguing that of blue, which would amount to restricting the main effectiveness of his stimulus to the region below the segment *ab*. The over-lapping effects produced by high intensity would make this seem very difficult, and possible only if his intensity near the violet limit of vision were exceedingly low compared to that of his other stimuli.

Segment *de* should be found from changes in sensation from blue to a yellowish, or possibly bluish, gray similar to the effect observed for *cd*; but the effect, if present, was not sufficiently well-defined for any of our observers to be measurable.

The yellow curves were of two sorts in the wave lengths longer than about 678 mμ. In one type, the yellow limit drops to the red in the region of 680 mμ, the initial sensation being yellow-red (orange) in that region, and then disappears. In the second variety, the yellow continues slightly above the red, often for an additional 15 mμ, or so, and there upon joins the red, the initial sensation being shades of yellow-red for wave lengths above this value to the visual limit.

Hess also noticed[41] the presence of yellow to the extreme red end of the spectrum.

The nature of the second type is quite clear, but the first leaves the completion of the curve in doubt; and since most of our observers gave results of the latter kind, it was felt desirable to attempt to locate this segment. One observer reported the appearance of a very slight yellow tinge in the red near the center of the retina with wave lengths in the region 670–680 mμ, thus determining approximately the upper wave length limit. But in most cases of this sort the red sensation so masked the yellow in the central region as to make determination impracticable. As a result of a fortuitous circumstance, the segment *fm* was plotted in one of these indeterminate cases. Curves had been obtained from the observer in question in which the yellow fell off into the red and disappeared at slightly above 670 mμ. The observer then contracted influenza and was ill for nearly a month. As soon as convalescence per-

[40] Op. cit., p. 14.

[41] Op. cit., Albrecht v. Graefe's Arch. f. Ophth., *35*, 4, pp. 17, 23; 1889; and Parsons, op. cit., pp. 40–41.

mitted, the measurments were repeated, with the result that the yellow curve previously determined was found to be practically unchanged, while the red fields were greatly reduced. In the region *fm*, the initial observation was now yellow-red, beginning along that segment. It was therefore concluded that the sensitivity to red had decreased sufficiently to produce a coincidence of limits in this region, the yellow effect having previously been masked or inhibited by the more intense red. The red curve determined before illness occupies approximately the position of the same curve in the figure. For the purposes of the present discussion it is unimportant which position of the curve is characteristic of the normal individual, since the limits of the sensitive areas vary so rapidly with stimulus intensity for a few degrees near the center of the field, that very slight changes in the ratios of chromatic sensitivity would account for such differences.

It has already been shown that the wave-length ranges indicated by these curves would not remain fixed, but would be broadened by increases in intensity. In regard to the range *jk* it is interesting to consider whether greatly increased intensity might not bring the curves for red and yellow together, or nearly so, at some large or moderate value of θ, the sensation below being yellow as a result of the overlapping fields analagously with the effect observed in the overlapping blue and yellow. Because of the yellow already present above, this would give the red and yellow curves the appearance of turning back upon themselves in approaching the axis.

5. THE "VIOLET" OR "REDDISH-BLUE" SENSATION

The curve labelled "violet" marked a consistent and fairly clear-cut change from a blue to a bluish-violet sensation. Because in the past many, but not all, observers have reported the violet as appearing reddish blue, the nature of the sensation giving this fifth curve of limits is uncertain. In the light of most of the literature, it appears probable that this curve does not represent the limits for a separate characteristically high-frequency sensation, but is a branch of the red curve, representing a second range of sensitivity. According to this view, the red curve became zero or sub-liminal at about 585 mμ, only to reappear in the vicinity of 460 mμ. This is in agreement with Maxwell,[42] who interpreted the change of hue in the lower part of the spectrum as a reddish tinge.

Dr. Ladd-Franklin was the most vigorous protagonist of the reddish-blue interpretation. She distinguished the spectral violet from purple

[42] Roy. Soc. Lond., Phil. Trans., *150*, 78; 1860.

only as reddish-blue from bluish-red.[43] However, in the experimental evidence she cites as "proof positive" there is no real question, for the pigments implied were capable of reflecting light in both the usual "Red" and "Blue" ranges of wave length. But the desideratum is the sensation resulting from a homogeneous stimulus in the short wave lengths.

It has already been mentioned that Judd found experimentally that in the wave lengths shorter than 460 mμ a shift in the hue toward the violet, produced by a decrease in the purity of the stimulus, could be counteracted by shifting the dominant wave length of the stimulus toward the blue. This would be conclusive evidence for the reddish-blue interpretation alone were it not for the presence in the white portion of the stimulus of wave lengths lying within the range of Curve I of Fig. 3.

There is no doubt that the effect of blue with a slight proportion of red is the color often called violet. But there is a question as to whether the effect, excited solely by the short wave lengths of the spectrum, whose limits were plotted in Curve V, is that of red in a second range of sensitivity or of another distinct primary color.

Parsons[44] seems to have seen red in the purple, but not in the spectral violet. Baird,[45] observed a violet quite different from purple. Of the latter, as the fovia is approached he says, "Beginning with yellowish, its tone gradually moved down the spectrum, passing through orange-yellow, yellow-orange, orange, orange red, red, purplish-red, and reddish purple, before the pure tone of the stimulus finally appeared." But "Violet appeared bluish, then blue, and took on a violet tint much later. Indeed, it sometimes happened that the violet tone of this stimulus was not recognized at ten degrees from the visual axis." While this evidence does not seem significant in deciding between reddish-blue and violet, it is a further substantiation of the observations for Fig. 3.

Burch's results already described make it appear that he succeeded in fatiguing the violet (or red) sensation without also fatiguing the blue, but this did not of itself prove the existence of a violet sensation distinct from red. A further fact which he considers as strong evidence for a non-reddish violet is the very great difference between the recovery intervals for red and violet. However, this would seem to the author to be vitiated by the differences in energy of the two parts of the spectrum. It is necessary merely to consider the quantum relation

[43] *The Physicist and the Facts of Color*, Science, *66*, 591; 1927.
[44] Op. cit., p. 27.
[45] Baird, J. W., Op. cit., p. 53.

$$h\nu = \tfrac{1}{2}mv^2$$

to conclude that greater changes might be expected in the violet than in the red, with a consequent longer time of recovery, even though the processes were the same. There is, however, one of his observations which impresses the writer as unfavorable to the conclusion that violet is no more than reddish-blue. This is the fact that upon fatigue for both red and violet, although the red sensation was still the first to recover, the effect of violet did not appear with the arrival of the blue sensation (at which time both the red and blue were operative)* but at a later time.[46] In order to account for this latter phenomenon satisfactorily, it seems that it would be necessary to consider Curve V as the result of a function quite distinct from Curve I.

The great importance attached in the four-color theories to the absence of a fifth sensation is in itself productive of a considerable predisposition in favor of the red. In this connection the fact that our observers, most of whom did not see red in the violet, were unencumbered by familiarity with any theory might be regarded as a point in their favor, partly offsetting their inexperience.

The only conclusion that seems to be justified at present is simply that Curve V represents the apparent limits for a process functioning chiefly within the lower blue, thus making the effect of short wave lengths definitely a complex one. Any decision as to whether this is a distinct violet process or a second range of sensitivity for red would be premature.

V. The Effect of Absorption by a Didymium Glass Filter

In order to observe the effect of variation of intensity by absorption and also to discover how much of the yellow response was produced by stray light, some measurements were made with a didymium glass filter in the beam from the grating.[47] The filter was in such a position as not to change the condition with regard to stray light.

The glass used was made by the Corning Glass Works; and in the absence of suitable equipment for measuring its transmission, it was

* From the conventional viewpoint it may be objected that the excitation of red by the short wave lengths is so slight as to remain sub-liminal. However, it must be remembered that the effect which distinguished "violet" from blue was observed to extend more than 13° (v. Curve V, Figure 3), and that the change in hue was well marked. It seems to the writer that a color whose sensitive area is as large as that of the non-blue component of the "violet" is not satisfactorily disposed of by merely assuming it to be sub-liminal.

[46] Op. cit., p. 10.

[47] Suggested by Dr. M. Luckiesh, Cleveland, 1931.

assumed to have substantially the characteristics shown by Curve I, which was obtained from measurements[48] of another sample of glass of the same manufacturer. Of course, I does not represent the intensity of the illumination in the present work, for there is still the distribution of intensity in the spectrum to be taken into account.

The results appear in Fig. 4, where the curves R_D, Y_D and G_D were obtained using the filtered light, and those without subscripts with the filter removed. All other conditions were kept constant as nearly as possible.

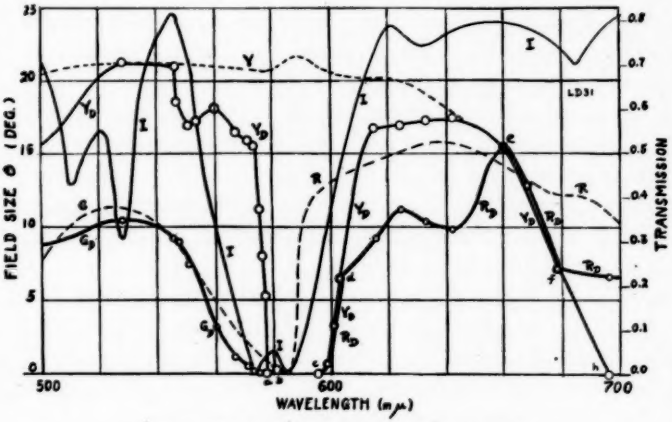

FIG. 4. *The effect of absorption by didymium glass.*

No color appeared between *a* and *c*, except at *b*, where there was a slight trace which may have been either yellow or achromatic. Along *cd* and *ef* the curves for yellow and red coincided, giving an initial sensation of yellow-red. The segment *fh* is doubtful, having been determined by the dubious point *h*, as explained in connection with *fm* of Fig. 3. There is an obvious shift of about 5 mμ between the limit curves and the Curve I, but it has not been determined whether this resulted from the disturbance of the apparatus while installing the filter or from a slight difference between the characteristics of the glass used and those of the measured sample.

However, the general shape of the field-size curves indicates clearly the effect of the decrease of stimulus intensity in the region of high absorption. The fact that the sensation drops to zero in the range of

[48] *The Ultra-Violet and Visible Transmission of Various Colored Glasses*, Gibson, Tyndall, an McNicholas, Bureau of Standards Technologic Papers, *148*, 9; 1920.

greatest absorption assures that the stimuli used in the measurements, particularly for the yellow, were derived from the spectrum and not from stray light. It is to be noted that the limits do not fall off in a constant ratio, which is a further substantiation of the work by Ferree and Rand. And here there can be no question of the result, for the limits were measured simultaneously for each intensity of stimulus. Finally, both the absorption and the elimination by the analysing slit of the "yellow" part of the spectrum failed to prevent other wave lengths from producing the yellow sensation, thus demonstrating again the production of the yellow color by wave lengths other than those usually thought of as xanthogenic.

SUMMARY OF CONCLUSIONS

1. The observations of the sequences, blue to yellowish gray to green, and yellow to yellowish gray to green in the range 480–495 mμ are believed to have the significance of showing the cancelling or complementary effect of two chromatic processes excited by the same physically homogeneous stimulus. In general, also, it is shown that (with the exception of ranges 460–480 and 580–585 mμ at the present low stimulus intensity) a stimulus homogeneous with respect to wave length ordinarily gives rise to more than one chromatic sensation in the central retina.

2. It has not been shown that five curves, and only five, can be plotted. However, an attempt to plot more than five leads to difficulties; e.g., the effort to secure a separate curve for orange yields a curve which is continuous with the red or yellow, or both. Similarly, while it is obvious that three curves would suffice for the interpretation of the results, the present procedure did not in any case give, directly, values for these curves. Apparently to do so would require considerable training of the observers before the criteria of consistence and continuity could be met; and it seems to the writer than this very fact may not be without significance.

3. A well-defined limit for a process here called "violet" has been found, without, however, determining whether it is to be considered as a separate characteristically short wave-length process or merely an additional range of sensitivity for red.

4. The physically mixed stimuli of Bull, Hess, Hegg, Baird, and Dreher, designed to give "stable colors", necessarily act to produce more than one of the chromatic processes whose limits are being observed. This conclusion follows from the premise that each of the component

stimuli of the mixture produces the effects observed for its particular wave length singly, so that the effect of the mixture is the resultant of all the simpler effects. The nature of the stimuli determines whether there will be blending, cancelling or inhibition, apparent dilution, etc., in various combinations, as a result of their simultaneous interaction, the complete processes being anything but simple. Even in the case of a complex stimulus which produces three simple chromatic colors under conditions such that two of them exactly cancel each other and leave the third (as in the "stable red" of the above authors), the single simple process is not functioning in isolation.

The writer is indebted to Dr. Deane B. Judd for valuable criticism and suggestions based upon a preliminary resumé of the paper. His sincere thanks are also due Dr. Samuel R. Cook, whose active cooperation made the research possible; and to Dr. M. Luckiesh, whose suggestions prompted the latter part of the investigation.

Statistical Data

Type of measurement	Number measured (eyes)	(subjects)	No. substantially agreeing with foregoing curves. (eyes)
Field sets for red, green, yellow, and blue	71	36	62
Field sets with violet included	18	10	15
Complete sets to check the five curves	10	5	10
Complete sets with didymium glass filter	4	2	4

Number of individual field measurements with homogeneous light

Red 776 Yellow 1,265 Green 788 Blue 722 Violet 383

Total giving positive results 3,934

RELATION BETWEEN CERTAIN TYPES OF LUMINESCENCE

By E. L. Nichols and Frances G. Wick
[Cornell University, Ithaca, New York. Received February 17, 1932]

The close resemblance of the luminescence due to exposure to the hydrogen flame (H-flame excitation) and of cando-luminescence in general to that produced by ultraviolet radiation and by cathodo-bombardment has been noted in various earlier studies. When H-flame excitation was discovered in 1921[1] the similarity as to color of the orange yellow glow of calcium oxide activated by manganese to the cathodo-luminescence of the same material was observed and H. L. Howes[2] demonstrated by means of a detailed spectro-photometric examination that the two spectra were essentially identical; being made up of the same sets of equidistant bands.

Where such activators as manganese, copper or bismuth are used, the luminescence spectra are broad banded and positive identification is only possible by such laborious spectro-photometric comparisons as that above cited. The rare earths, on the other hand, when employed as activators, give comparatively narrow, well defined bands which can be located at a glance with a simple, low dispersion spectroscope. These bands are characteristic of the activator and have long since been comprehensively mapped by Urbain[3] and others for the spectra of various materials subjected to cathode rays.

In the course of observations of the cando-luminescence of incandescent beads,[4] by a method developed from that of Lenard,[5] it was noted in 1920, that when beads of borax, sodium phosphate or the like, containing a rare earth were heated in the hydrogen flame they glowed strongly at certain critical temperatures. When viewed with the spectroscope, preferably while cooling, bright bands in the spectrum flashed out as the bead passed through the particular temperature in question, and since this so called "Transformation Spectrum" coincided, as to its bands, with that given by Urbain for the cathodo-lumi-

The senior author wishes to make acknowledgement to the Heckscher Research Council of Cornell University for assistance rendered in support of this research.

[1] Nichols and Wilber, Phys. Rev. (2) *17*, 453–468; 1921.

[2] Howes, Phys. Rev. (2) *17*, 469–474; 1921.

[3] Urbain, Comptes Rendus, 143, 825; 1906. Little, Textbook of Inorganic Chemistry, Vol. IV, p. 290.

[4] Nichols and Wilber, Proc. Nat. Acad. Sc., *6*, 693–000; 1920. Phys. Rev. (2) *17*, 707–717; 1921.

[5] Lenard, Ann. der Physik, *17*, 199–247; 1905.

nescence of the activator it was inferred that in cando-luminescence and in cathodo-luminescence we have the same process. It is the purpose of this paper to describe further experiments in confirmation of this view.

Oxide of thorium, calcium or lanthanum, respectively, with known amounts of rare-earth elements in solid solution some of which had been prepared by Dr. L. J. Boardman[6] in 1929, were used in these comparisons.

They were subjected in turn to:

(1) H-flame excitation on the copper block following essentially the procedure developed in 1927.

(2) Excitation by ultraviolet radiation (Fe spark through red purple corex glass No. 2507, 4.5 mm thick which transmitted to .27μ)

(3) Cathodo-bombardment.

The color of glow and spectral components produced by these three independent modes of excitation were compared. The results are given in Tables 1 to 6.

SUMMARY OF OBSERVATIONS

TABLE 1. *Aluminum oxide activated by terbium.* (Preparation by Boardman.)

Excitation	Cathode	H flame	U.V. spark (after exposure to H flame)
Color	green	green	green
Spectrum	0.610 d Eu 0.590 d Tb 0.550 m Tb 0.542 str. Tb 0.520 d Eu 0.510 d Eu 0.485 m. Tb	 0.550 0.542 0.490–0.480	 0.550 0.542

Bands in the cathode ray spectrum were comparatively sharp, narrow and easy to locate. Only the stronger of these were duplicated in the other two spectra and in these no bands not in the cathode spectrum were visible. The color of the luminescence was appreciably identical for all three modes of excitation but when the oxide under the H-flame was heated to about 500° this green glow was supplanted by a deep red luminescence and a band between .640μ and .620μ replaced the bands noted above.

[6] Nichols and Boardman, J.O.S.A., *20*, 115–130: 1930.

Luminescence under spark excitation was bright enough to permit location of bands (.550μ and .542μ) only *after* exposure to the H-flame; an effect discussed further in a later paragraph of this paper. In the

TABLE 2. *Thorium oxide activated by terbium.*

Excitation	Cathode	H flame	U.V. spark
Color	green (brilliant)	green followed by red at higher temperatures	green (very dim)
Spectrum	0.630 d Sm 0.615 d Eu 0.590 sh Eu 0.575 sh Dy 0.550 str Tb 0.542 v str Tb 0.490–480 Tb	0.630 0.550 str 0.542 v str. 0.490–480	 0.550 d.

case of dysprosium, Table 3, the dim glow under spark excitation was *not* appreciably enhanced by previous exposure to the H-flame. The instability of this material was such as to make comparisons difficult and uncertain.

TABLE 3. *Thorium oxide activated by dysprosium.*

Excitation	Cathode	H flame	U.V. Spark
Color	Yellowish green	green, then red*	
Spectrum	0.670 m.....Dy 0.600 d.....Eu 0.590–0.582...Dy 0.580–579v str Dy 0.550–540 m...Tb 0.492–470sh on Dy side of longer wave-lengths (crest 0.490)	0.670? vvd 0.550–540 (crest) 0.500–.475	0.650 Eu 0.600 0.580–575 0.550–540 0.500 (diffuse)

* Under *flame* excitation there was a brief glow, *green* in color, during which the bands recorded in the table flashed up and disappeared. This was followed by a *red* glow the spectrum of which consisted of two new bands i.e. 0.650μ–0.600μ (Eu or Sin); 0.525μ–0.500μ (Eu).

In the cathode tube it soon became inert, the very strong dysprosium band between .580μ and .570μ being the first to disappear. The color of the glow shifted, as this band faded, from yellow to green. Samples

which had been in contact with the H-flame glowed greenish in the cathode tube, the spectrum was momentarily brighter and sharper; the red *dysprosium band* (.670μ) *was now the first to disappear.*

Under the spark there was almost complete inactivity, unless the material had been in contact with the flame previously, in which case the spark excitation produced a greenish glow of sufficient brightness to permit of the location of the bands noted in the table.

The only sample of dysprosium oxide available for these observations was of doubtful purity and the bands recorded in the cathode spectrum indicate the presence of terbium and europium in addition to dysprosium (See Table 3).

TABLE 4. *Lanthanum oxide activated by europium.*

Excitation	Cathode	H flame	U.V. spark
Color	red	red	red
Spectrum	0.630 Sm*	0.630	0.630
	0.618 Eu	0.620	
	0.593 Eu		
	0.583 Dy		
	0.554 Tb+Eu	0.550	0.550
	0.539 Tb	0.539	
	0.512 Eu	0.510	0.500 Dy
	0.493 Dy		(broad diffuse)
	0.471 Dy		

* This band which occurs in Tables 2 and 4, is tentatively assigned to samarium since it coincides with Urbain's Sm band of that wave-length. It falls within the Europium group, however, extending from 0.637μ to 0.635μ and may be a composite of 0.625μ and 0.637μ (Urbain).

TABLE 5. *Thorium oxide activated by neodymium.*

Excitation	Cathode	H flame	
Color	purplish fades rapidly	yellow, then red	
Spectrum	0.590–0.575 Dy	0.575	0.575–0.500
	(crest at 0.580)	(ill defined)	(diffuse)
	0.550 d Tb		
	0.520 d Eu		
	0.490–0.480 Dy		

The determination of the elements to which these often broad and

ill defined bands should be assigned is greatly facilitated by the fact that the bands of the rare earths, as given in Urbain's tables,[8] tend to occur in groups with but little troublesome overlapping. The bands listed above are too dim and ill defined to afford an altogether satisfactory comparison but it seems reasonable to assign them to dysprosium, terbium and europium respectively.

Undiluted neodymium oxide, when heated in the Bunsen or O-H flame, glows with the striking banded spectrum first described by Bunsen and Bahr in 1864. With the spectrophotometer[7] ten components were located in this spectrum in 1922 six of which may be assigned to dysprosium and three to samarium with one band in the far red unidentified. The bands are visible at 600° and disappear just above 1000° with a well defined maximum at 700°.

When neodymium is dissolved in a bead of sodium phosphate ($Na_4P_4O_{11}$) heated in the H-O flame, similar bands flash out and six of these were located with a hand spectroscope in 1925.[8] They seem to belong to samarium, europium and dysprosium.

Neodymium, in general, seems to be a carrier for other rare earths to which the luminescence is due. Neither Urbain[3] nor Tanaka[9] found neodymium bands except in the violet and ultraviolet. Bauer and Marc[10] record four bread bands three of which probably indicate Europium. Our Table V, above, denotes the presence of dysprosium, terbium, and europium as activating impurities.

Praseodymium, like dysprosium, when in solid solution, is unstable in the H-flame and under cathode bombardment. In the presence of other rare earth activators its bands tend to be suppressed.

TABLE 6. *Calcium oxide activated by praseodymium.*

Excitation	Cathode	H flame	U.V. spark
Color	Bluish	Orange	Orange-red
Spectrum	0.640–0.630 Sm	0.630	0.650
	0.620 Eu		0.620
	0.580–575 Dy		
	0.550 -545 Tb		0.550–547
	0.530–510 Eu	0.530	

[7] Nichols and Howes, Phys. Rev. (2) *19*, 300–318; 1922.
[8] Nichols and Howes, J.O.S.A., *13*, 576–586; 1926.
[9] Tanaka, J.O.S.A., *8*, 287–318; 1924.
[10] Bauer and Marc, Berichte d Chem. Ges. *34*, 246; 1902.

In Thorium Oxide it responded very dimly to cathode and spark excitation. Under the action of the H-flame the glow was red, then green. The spectrum indicated the presence of europium dysprosium, thorium and samarium, but all praseodymium bands were suppressed.

In Calcium Oxide (after exposure to the H-flame), the bands observed were those recorded in Table 6.

THERMOLUMINESCENCE

Observations were also made to determine whether these substances, like certain fluorites containing traces of rare earth elements, were rendered thermoluminescent by previous exposure to X-rays.[11] The specimens were exposed for 20 minutes at a distance of 30 cm to X-rays from a tungsten target tube running on about 75,000 volts and 4 milliamperes. Our materials were, with a few exceptions, nearly non-thermoluminescent after such exposure. The exceptions were:

Aluminum Oxide with Terbium; thermoluminescence after exposure to X-rays, dim green; color similar to that produced by excitation with H-flame, cathode bombardment or spark.

Calcium Oxide with Praseodymium; very dim thermoluminescence after exposure to X-rays.

Calcium Oxide with Samarium; This substance was studied with especial reference to the relative and combined effects of X-rays and spark. The results were as below.

Calcium oxide with samanium.

After exposure to	Thermoluminescence
(a) Spark	Orange-green, then green, very dim.
(b) X-rays (15 minutes)	Orange-green, not very bright nor of long duration
(c) X-rays, then spark	Orange-green, then green; much brighter and of longer duration.
(d) Spark, then X-rays	Brightness and color similar to (c). Order of exposure appears to make little difference.

In a similarly preliminary and tentative manner the possible influence of the H-flame on other modes of excitation was investigated and it was discovered that previous exposure to the active zones of such a flame even under conditions which precluded any considerable rise of

[11] Wick, J.O.S.A., *14*, 33–44; 1927.

temperature (i.e., by applying the flame to a very thin layer of the material which was in intimate contact with the massive copper block) produced a marked effect.

Thus:

Aluminum oxide activated by terbium.
Lanthanum oxide activated by europium.
Thorium oxide activated by praseodymium.
Calcium oxide activated by praseodymium.

responded to spark excitation only after exposure to the H-flame.

The response under the spark of *thorium oxide activated by dysprosium* was greatly enhanced by previous treatment with the flame as was also the cathodo-luminescence of that substance. On the other hand, the very dim luminescence under the spark of *thorium oxide activated by terbium* was not increased perceptibly by treatment with the flame while the rather strong purple spark luminescence of *fluorite from Weardale England*[12] which had been previously pulverized and heated to remove its natural thermoluminescence and which then responded with a purple glow under the spark, became quite inert to such excitation after exposure to the H-flame.[13]

The foregoing observations establish the existence of effects which demand further study. They also indicate a relation between the processes involved in the various modes of excitation which is very pertinent to the theme of this paper.

TEMPERATURE CRESTS

Marked accessions of brightness in the luminescent glow at definite temperatures has recently been recorded in the thermoluminescence of fluorites containing rare earths[14] a fact which has led us to study the variations of brightness with temperature of certain of the substances under investigation in the present paper. The materials selected for this purpose were:

Aluminum oxide activated by terbium.
Thorium oxide activated by terbium.
Lanthanum oxide activated by europium.

Samples of these were mounted on the copper block in the manner already described, and were gradually raised to a temperature of about

[12] For further work on this material see Wick, Phys. Rev., *24*, 272–282; 1924.

[13] It was noted by Przibram in his studies of thermoluminescence that light was destructive of the thermoluminescence of substances previously exposed to radium.

[14] Wick, J.O.S.A., *21*, 223–231; 1931.

600° by heating the block from below and the character of the glow excited by exposure to the H-flame during this heating process was

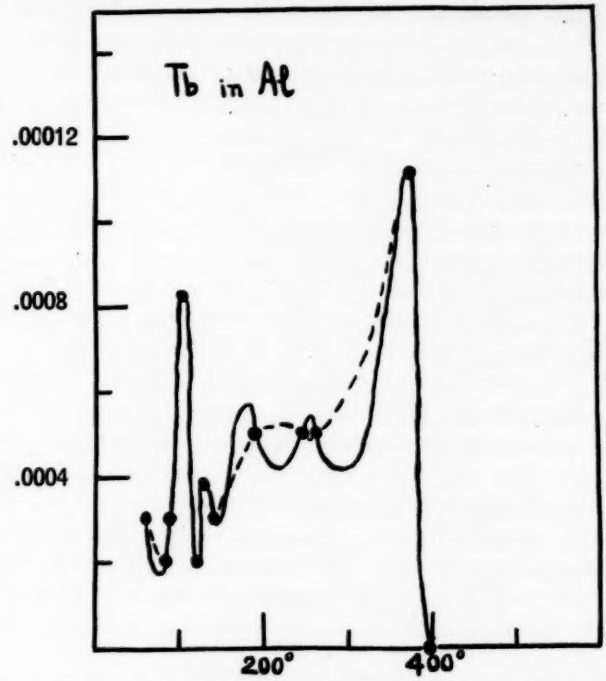

FIG. 1. *Aluminum oxide activated by terbium. Cando-luminescence excited at different temperatures by the hydrogen flame.*

TABLE 7. *Aluminum oxide activated by terbium.*

	Temp. C	Brightness cp/cm²		Temp. C	Brightness cp/cm²
A red glow just visible at 477°	398°	0.0000724		133°	0.000389
was replaced by green at 457°.	378°	0.00110		124°	0.000209
The latter persisted at all lower	267°	0.000507		108°	0.000828
temperatures with slight changes	253°	0.000507		92°	0.000302
of hue.	182°	0.000502		87°	0.000209
	145°	0.000302		67°	0.000302

noted. An optical pyrometer was now sighted upon the glowing oxide, as in previous determinations of temperature crests at higher temperatures. The block was allowed to cool slowly during which time the oxide was continuously under excitation by the flame and a succession of

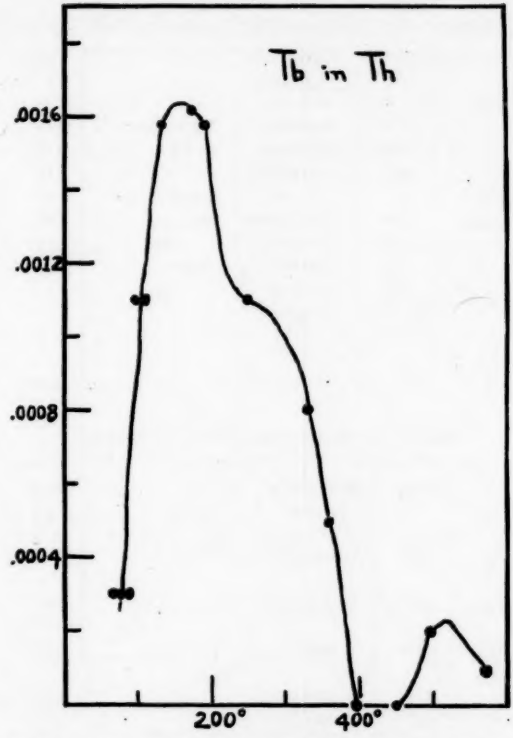

FIG. 2. *Thorium oxide activated by terbium. Cando-luminescence excited at different temperatures by the hydrogen flame.*

brightness readings was made. Temperatures were those of the surface of the copper block with which the thin layer of oxide was in intimate contact.

The results of these measurements are shown in Tables 7, 8, and 9 and in Figures 1, 2, and 3.

The minor crests in the curves in Figs. 1, 2 and 3, may be regarded as subject to subsequent verification. Such sudden variations of brightness are, however, not uncommon at higher temperatures. Compare,

for example, the crest at 1400° (of Al_2O_3 with an unknown activator); that at 1075° (of Th in Nd) mapped in 1930.[15] Figure. 2 and 3 show similar temperature crests for thorium oxide activated by terbium and lanthanum oxide activated by europium.

TABLE 8. *Thorium oxide activated by terbium.*

	Temp. C	Brightness cp/cm²		Temp. C	Brightness cp/cm²
	571°	0.000101	Main crests	175°	0.00162
Red glow	488°	0.000209	at 150	135°	0.00158
	447°	0.0000083		110°	0.00110
Red vanishes and			Green glow	98°	0.00110
is replaced by green	398°	0.00000095	persists.	92°	0.000302
	375°	0.000502		74°	0.000302
	343°	0.000828		75.5°	0.000302
Green glow	253°	0.00110			
	195°	0.00158			
Minor creasts at 250–260 (See Fig. 2)					

TABLE 9. *Lanthanum oxide activated by europium.*

Remarks	Temp. C	Brightness cp/cm²		Temp. C	Brightness cp/cm²
Ruddy glow just visible at 530°	495°	0.0000288		230°	0.000252
This color persists throughout the	447°	0.0000912		191°	0.000159
temperature range to 70°.	405°	0.000251		139°	0.000159
Minor crests are	375°	0.000209		141°	0.000209
indicated at 400,	337°	0.000214		115°	0.0000661
320, and 170.	323°	0.000501		107°	0.0000219
	298°	0.000458		88°	0.0000502
Main crest at	263°	0.000742		70°	0.0000417

The temperature crests depicted in the present paper are shown because they serve to connect the phenomena of H-flame excitation with thermoluminescence where similar crests have already been described[14] and with the luminescence of incandescent solids.

[15] Nichols, J.O.S.A., *20*, 106–114; 1930.

TENTATIVE EXPLANATION OF THE RELATION BETWEEN MODES OF PRODUCING LUMINESCENCE.

Reasons for regarding luminescence in general as an accompaniment of oxidation or of an equivalent process have been presented in an earlier communication.[15] That this is true in bio-iluminescence has long since been established; that it is true in certain inorganic cases such as

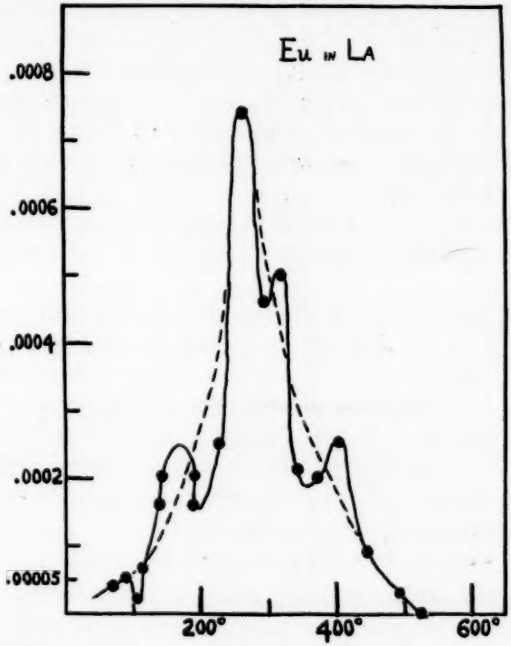

FIG. 3. *Lanthanum oxide activated by europium. Cando-luminescence excited at different temperatures by the hydrogen flame.*

the glow of phosphorous is obvious. The production of an easily oxidizable substance (i.e., by reduction) and the presence of free or easily available oxygen or its equivalent are essential. These are commonly the direct result of a simple oxidation-reduction process and the luminescent glow may follow immediately from the recombination of the separated components.

Such spontaneous luminescence is prevented if, by any means, the supply of free, active oxygen is depleted. This may be by chemical action, e.g. in the experiment of Dewar who noted that the glow of fluorescent solutions was inhibited by hydroquinone.

The bodily removal of the free oxygen has a similar result. Thus, in the study of the cathodo-luminescence of the powdered oxides used in the present investigation it was found that the glow of the bombarded material vanished as evacuation progressed; an effect previously observed in the case of similar substances and definitely connected with the depletion of the oxygen within the vacuum tube.[16] Oxygen from without, entering the tube, restored the glow, just as oxygen (in active form) applied to a substance previously exposed to X-rays, has been found to produce luminescence.[17] In short, cathode rays, X-rays, ultraviolet radiation (as from the spark) and the H-flame all tend to reduce the class of substances capable of luminescence. *The glow may be due to subsequent reunion with oxygen thus set free in active form or with oxygen derived from another source.*

The various methods of excitation being similar and, indeed, almost identical, are therefore supplementary and, to some extent, interchangeable.

Previous treatment by the H-flame may thus greatly enhance the glow produced by the spark or by cathodo-bombardment as already described on a foregoing page. Exposure to the spark and then to X-rays or vice versa may increase the thermoluminescence of a substance above that due to exposure to X-rays alone. Under other conditions, not as yet fully determined, the one process may forestall or supplant the other with consequent inhibition of the effect.

[16] Nichols and Wilber, Phys. Rev. *17*, 707–717; 1921.
[17] Nichols and Wick, Nat. Acad. Sc. Proc., *16*, 32–36; 1930.

TRICHROMATIC FUNCTIONS OF THE AVERAGE EYE*

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[Department of Physiology and Pharmacology and the Department of Psychology, University of Louisville, Kentucky. Received January 11, 1932]

1. Twofold Aim of the Work

This work deals essentially with two problems arising from the trichromatic theory of color vision. The first is to determine the amounts of three selected primary hues required by the average eye to make a heterochromatic match to monochromatic and to white light. It is a matter of common observation that even normal individuals differ with each other as to these amounts. Yet each investigator has offered results for only very few subjects, Wright (11) having dealt with the largest number, ten. We have, therefore, attempted to find the range of these individual differences, their statistical significance, and tentatively to account for them. For this purpose we have employed 68 subjects (66 medical students and the two authors). This number is sufficiently large to represent the population in general, and to give data for the characterization of the average eye.

The second problem is to determine exactly where the loss of saturation of mixtures begins, and to account for this peculiarity. This loss of chroma has been reported, beginning with Maxwell (cf. 1, p. 227) and with consistency by König and other subsequent investigators, for extensive regions of the spectrum. We also find a loss of chroma, but for a far more restricted region of the spectrum. Certain explanations have suggested themselves of the difference in our results, which lead to the tentative formulation of hypotheses to account for the causes of desaturation.

2. Apparatus

The apparatus, which is an adaptation of the earlier optical systems of Maxwell (cf. 1, p. 223) and of Abney (1, p. 38) designed for the same purpose, is so arranged that the subjects may mix selected spectral lights in any proportion in the lower half of a circular field seen through a tube, and can bring this mixture to match the upper half of the field. The two semi-circular fields meet sharply without overlapping. Fig. 1 is a diagram of the set-up. The upper field may be illuminated with white light, or with a single spectral light of any wave length, or with a

* This work was made possible through a grant from the funds of the American Association for the Advancement of Science.

mixture of one spectral light of any wave length together with white, —the latter serving to desaturate. It is essential to note that in our work, unlike that of Abney, the stimuli consist of transmitted light, and not of reflected light from a receiving surface.

The optical system, of which the description follows, was completely enclosed and entirely free from scattered light and uninfluenced by the surrounding illumination of the laboratory. When the illumination for either field was cut off, it became completely invisible though the other might have been maximally illuminated. This shows that the light for

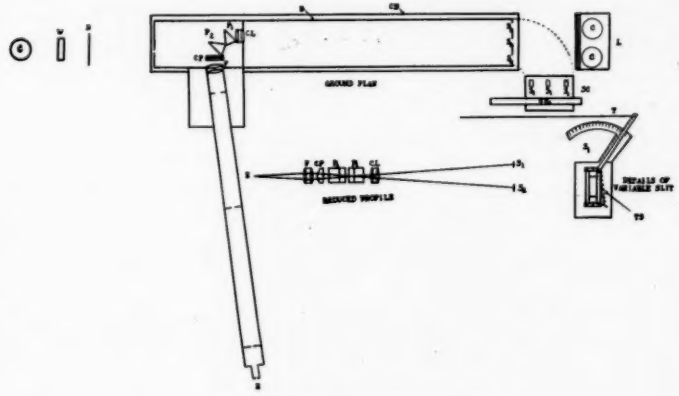

FIG. 1. *The colorimeter apparatus.*

each field was properly segregated in order to prevent scatter within the optical system. The whole circular stimulus field subtended an angle of 1° 40′.

The optical parts are arranged as in Fig. 1. The C on the extreme right is the light source for the spectral hues. Each S_1 is a variable slit to supply variable intensities of a given primary to the lower half of the field already mentioned. S_2 is a slit of constant width supplying any one selected wave length to the upper half of the field to be matched. CL is the collimator lens, P_1 and P_2 equilateral prisms, CP a biconvex prism, F focusing lens, E eyepiece aperture, ½ mm wide and 2 mm high; CB rigid cast-iron base for carrying the entire optical system. An important feature of the apparatus is that all the slits have the same source of illumination which lights the entire surface of the slit-carrier homogeneously. Hence the relation of the slits to the light source is constant, so that an accidental variation in the source will cause similar and simul-

taneous variations in the upper and lower slits. From this it is clear that the slit widths required in making a match are the only quantities which need be measured. It was at all times possible, therefore, to check up the correctness of the relationship between the upper and lower slits, by showing that for the same spectral color equal widths of the lower slit and of the corresponding upper slit gave a perfect match of the two halves of the circular field.

The light source C at the extreme left is an auxiliary illuminant designed to deliver white light to the upper field whenever necessary. It is a 100 watt Mazda, on the same circuit as the main illuminant. W is a Wratten daylight filter no. 78 AA which changes the tungsten light to standard white. D is a diffusing surface of double ground white glass, 35 cm in front of C. At the extreme right of Fig. 1 is the principal illuminant, acting as the source of the individual monochromatic lights employed in making the matches. It consists of a ventilated tin box on which the front face carries three sheets of white optical glass (Eastman) ground on both sides and held about 3 mm apart. In the box are two 500 watt Mazda flood lights. The current to these is controlled by a rheostat and voltmeter, so that the voltage remains at 97, which according to Forsythe (2) causes the lamps to burn at a color temperature of 2790 K. This temperature cannot be said quite to govern the distribution of light from which the stimuli were ultimately taken, because the white optical glass can be seen to tinge the light slightly yellowish.

Each of the slits S_1 may be so placed that it transmits any one desired wave length. By means of the thread T, working against the tension spring TS each of the S_1 slits may be widened or narrowed from the subject's seat, so that either he or the experimenter may regulate the composition of the mixture. For each S_1 slit there is a separate control thread. The design of the slits and the method of their attachment is similar to that described by Abney (1). The arm for varying the slit-width and the scale for reading the latter has been added. The scale has been calibrated with a micrometer microscope.

The wave-length scale which lies horizontally along the slit-carrier enables one to select such a position for each upper and for the lower slit that any desired wave length may be transmitted to the eye. This scale was calibrated by means of the Gaertner wave-length spectrometer.

The lenses, achromatic doublets, focal length 1 M, were furnished by Gaertner, as were the two 60 degree crown prisms.

The biconvex prism CP serves to direct the light from the upper

slits and the lower slit into the focusing lens F, which brings the rays to a focus in the eyepiece aperture E. The white light from the auxiliary illuminant C, comes in, when required, at the same angle from the horizontal as does the light from the lower slit S_2. It is reflected from the face of prism P_2 and refracted unanalyzed by the upper part of the biconvex prism CP into the focusing lens F, and thence into the eyepiece aperture.

In practice the apparatus was set up so that the lower field was illuminated by a mixture of lights coming from one or more of the slits S_1. These slits were placed to transmit to the eye the wave lengths 670 (red), 517[1] (green), and 460 or 480 (blue).

Three slits were used in the lower-variable wave-length slit-carrier. These were each of constant width. The narrowest, 0.42 mm wide, was used in transmitting wave lengths 620, 600 and 580. The width of the medium slit was 1.22 mm. It was used in transmitting wave lengths 560, 540 and 520. The third slit, 2.68 mm wide, was used for wave lengths 500, 480 and 460. The slit-width measurements reported here are all reduced to compare with the narrow slit, thus the measurements taken with the narrow slit are reported as taken, those with the medium slit are multiplied by 0.345 and those with the wide slit by 0.157 before being reported. The upper field could also be illuminated by white light from the auxiliary illuminant C.

3. EXPERIMENTAL PROCEDURE

The first step was to make a match between white and a mixture of the three spectral primaries 670, 517 and 480. The experimenter made an approximate match for his own eye. The subject was thereupon requested to look in, holding his eye so that the beam from the eyepiece aperture passed through the center of his pupil to eliminate intraocular chromatic aberration and to judge whether the mixed field was different in color or brightness from the white field in the upper half. With the subject resting, adjustments were made of the slit-widths in accordance with his answers, until he was satisfied, after resting each eye in turn, that the match was perfect as to color and brightness. The same procedure and safeguards were used in making dichromatic mixtures to match a monochrome.

We must emphasize the fact of light adaptation in the procedure just described, and that all the data secured from our experiments refer to the light-adapted eye, and to the center of the fovea.

[1] The symbol $m\mu$ is omitted here and hereafter.

4. FORM OF PRESENTATION OF RESULTS

Since in a great many treatments of color-mixture data the emphasis has been on the relative merits of various kinds of graphic forms and algebraic transformations, it is well to make clear that we have regarded these issues as secondary. Our aim has been to present the results in as physiologically realistic and significant a fashion as possible. To that end we have found it advisable deliberately to disregard certain conventions which have been employed in the treatment and presentation of spectral mixture data since the time of König. The reasons for these conventions have been mathematical expediency and logical nicety rather than physiological realism. Instead therefore of recalculating the values to conform to assumptions of equality of chromatic valence of the three primaries, the slit-width of each primary required for a white, or any other match, is presented and plotted directly as found.

It will be noted that all values in Tables 1–4 which represent individual distribution of sensitivity vary by 0.05 mm of slit-width. This

TABLE 1. *Showing in mm of slit-width the distribution among 68 subjects of the amount of the 670 (red) primary required in the match to white.*

2.30	
2.25	A,
2.20	
2.15	H,
2.10	
2.05	Z′
2.00	
1.95	E Z E′ F′
1.90	I B′ C′ H′ N′ W′ X′ Y′ K,
1.85	
1.80	B G C, I, S,
1.75	H T U A′ D′ G′ O′ P′ Q′ V′ B, D, E, M,
1.70	D F Q R I K L M R F U J L N T,
1.65	
1.60	C J K L M N O P Y S G
1.55	A W
1.50	S V O
1.45	
1.40	
1.35	X
1.30	
1.25	
1.20	
1.15	
1.10	T

(Average slit-width for the 68 subjects, 1.74 mm)

is a simplification resulting from placing each actual reading in the nearest group.

5. DISTRIBUTION OF R, G, AND B SENSITIVITY IN THE POPULATION

Using the standard white light already referred to in the description of the apparatus, each of the 68 subjects made a match with a mixture of the three selected primaries 670, 517 and 480. The results are presented in Tables 1–3. The values are slit-widths in mm, and each letter

TABLE 2. *Showing in mm of slit-width the distribution among 68 subjects of the amount of the 517 (green) primary required in the match to white.*

2.05	T′
2.00	
1.95	
1.90	
1.85	
1.80	
1.75	
1.70	
1.65	P
1.60	X
1.55	N
1.50	G E′ N′
1.45	
1.40	H A′ G′ H′ M′ O′ P′ Q′ S,
1.35	C D E F Q R I K′ L′
1.30	B J K L O T V W Z C′ D′ F′ R′ G, J, L, O, T,
1.25	
1.20	I S U Y S′ U′ V′ W′ Z′ A, B, C, E, F, I, M,
1.15	M B′ D, H, K, N,
1.10	X′ Y′
1.05	
1.00	A

(Average slit-width for the 68 subjects, 1.31 mm)

represents one subject. Each subject is represented by the same letter in the white and chromatic matches. Thus the reader may trace a given subject's consistency and sensitivity to a certain primary by finding his letter in Tables 1–3 for white and in Table 4, the summary chart, for color matches. For example, for subject *S*, (one of the authors), Tables 1–3 show that to match white he required 1.80 mm of red, 1.40 of green, and 1.30 of blue. The summary chart in Table 4 shows that to match 620 he required 2.10 mm of red, and 0.25 of green, etc. The averages of results in Tables 1–4 are given in Table 5. The curves in Fig. 2 have been plotted for all the values in Table 5, except those of white.

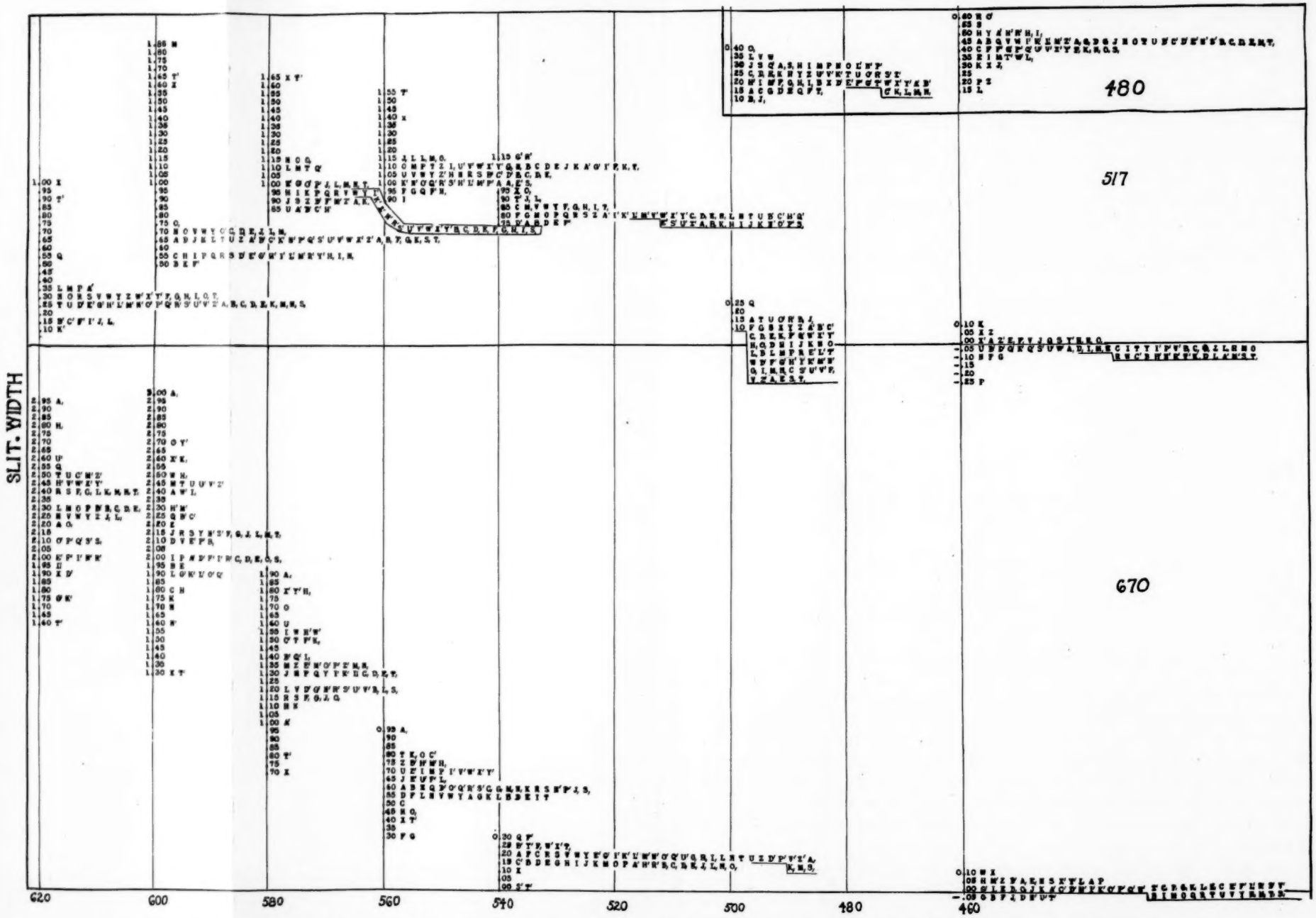

TABLE 4. *The summary chart showing, in mm of slit width, the distribution among 68 subjects of the amounts of primaries of 670,517, and 480 required in matches to monochromes.*

TABLE 3. *Showing in mm of slit-width the distribution among 68 subjects of the amount of 480 (blue) primary required in the match to white.*

2.20	L Z
2.15	E
2.10	
2.05	
2.00	
1.95	
1.90	
1.85	C T U
1.80	
1.75	B
1.70	
1.65	I P
1.60	K S′ Z′ A, B, C, D, E,
1.55	C′ R′ F, T,
1.50	D G′ L, M,
1.45	
1.40	A Y U′ V′ W′ X′ Y′ G, J, K, N, O,
1.35	A′ D′ E′ F′ H, I,
1.30	F R N′ S,
1.25	
1.20	N Q O′ P′
1.15	V W X B′ I K′ L′ M′ Q′
1.10	G M
1.05	
1.00	O
.95	S
.90	J T′
.85	
.80	
.75	
.70	
.65	H
.60	
.55	H′

(Average slit-width for the 68 subjects, 1.38 mm)

The distribution in the population of sensitivity in the white and chromatic matches, as shown by Tables 1–5 reveals a significant contradiction to König's declaration (6, p. 256) that the population consists of two distinctly separated groups which would form a bimodal sensitivity curve, with the normal trichromats in one group and the anomalous in another. This assertion, true enough apparently of his four subjects, is statistically questionable as a generalization from so few data, and seems inconsistent with the results obtained from a large group such as ours. Among our 68 subjects we find no such segregation into bimodal groups, cf. 15, 16, 17, 18, 19. There is simply a normal dis-

TABLE 5. *Data from which the curves of Fig. 2 have been plotted showing average slit-widths for matches of all subjects. Values calulated by averaging the data for 68 subjects in Tables 1, 2, 3, 4.*

	Red (670)	Green (517)	Blue (480)
460*	.007	−.040 (.000)	.423 (.383)
480*	—	— (.040)	.420 (.380)
500*	—	1.07 (.130)	.233 (.210)
517	—	.420	—
540	.182	.822	—
560	.618	1.070	—
580	1.320	.985	—
600	2.130	.672	—
620	2.270	.289	—
670	.420	—	—
White	1.740	1.310 (1.441)	1.380 (1.249)

* The arithmetical manipulation by means of which the values in the parentheses were derived and plotted is explained in the section "Peculiarities of Mixtures below 517."

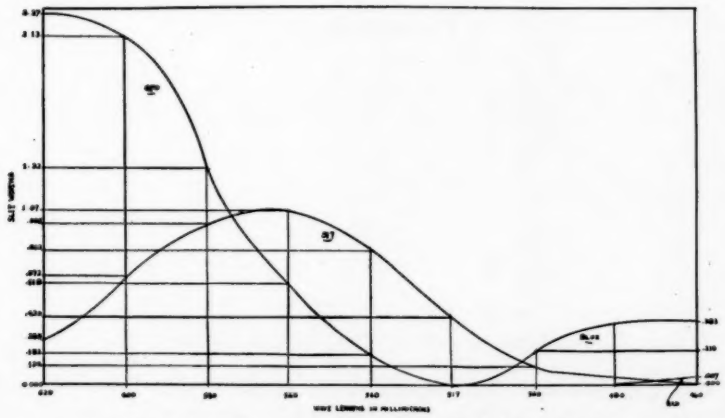

FIG. 2. *Excitation curves for average eye, for data in Table 5.*

tribution with a distinct massing about the median. In any discussion of the chromatic functions of the average eye, and in the establishment of colorimetric standards, this fact should be of considerable importance.

The distribution of amounts necessary for a match, as shown in Tables 1–4, may be safely regarded as a difference in the functioning of the visual mechanism from subject to subject, that is, as a genuine individual difference of sensitivity to any given wave length. The fact

that the two authors (W. F. H., represented by T_2, and E. F. by S_2) found it easy to reproduce their own previous matches at different sittings after an interval of weeks; that their matches were unsatisfactory to subjects who required either more or less of red or green; and that the matches of these subjects were acutely unsatisfactory to the authors, indicates that the distribution of results is not the consequence of mere experimental error. Rather it is due to a fundamental and real physiological difference of function from individual to individual.

Although there is no question that the distribution represents genuine differences in physiological functions, it is difficult to select the precise structures in the visual mechanism to which to ascribe these differences. Two possibilities suggest themselves as the most likely. The differences shown in Tables 1–4 might be due to differences in the sensitivity of receptors, or to absorption of different amounts of light by the pigment lying in front of the sensitive retinal elements of different eyes. Since the macular (6, p. 346) and lenticular pigments absorb blue light more than light of longer wave lengths, they cannot be used to explain fully the wide deviation from normal of the red and green values of subjects whose blue values seem normal. There must, apparently, be a decided individual difference in the sensitivity of the red and green receptors which cannot be related to the amount of macular and other yellow pigment.

Another observation that contributes evidence in favor of this view is that when the authors presented their white match to the subjects, the latter might say that it appeared too red or too green. It was unusual that it appeared too yellow or too blue. Furthermore, when the adjustments of slit-widths were made to the satisfaction of the subject the trichromatic field usually seemed to the authors to vary toward the red or the green, and rarely toward the blue or the yellow. If differences in density of macular pigmentation played a preponderating part in causing the differences between individuals, the standard white would appear to the eye with less pigment as bluer, and to the eye with more as yellower. The trichromatic light would, on the other hand, reflect these changes to a lesser degree, since the blue here is of relatively long wave length and hence is less absorbed than the average blue of standard white. The variations in the blue given in our distribution tables is to a large extent the result of the coarseness of response to blue. This coarseness of response to blue may be the reason why Wright (11) and Sinden (9) publish trichromatic and complementary whites, respectively, varying in the blue-yellow direction. They attribute this variation to differ-

ences in the amount of pigmentation. In view of the coarseness of the blue response, we should hesitate to accept the pigment as the sole, or even the principal cause.

6. THE LOSS OF SATURATION (CHROMA) IN SPECTRAL MIXTURES

A. RELATION OF DARK-ADAPTATION TO LOSS OF SATURATION

König (6, p. 256) notes that in a very extensive region of the spectrum the mixture of two lights to match an intermediate monochrome loses saturation as compared to the monochrome. In our work no loss of saturation of mixtures was observed above 517, and at 500 was barely perceptible. It must be emphasized that our observations were made with the *light-adapted* eye. How was it with König? Nowhere does he make an explicit statement, but there is evidence to support the inference that his work was done in the dark, and, therefore, with a more or less dark-adapted eye. The evidence for this inference lies in the nature of his color-mixing apparatus, and particularly in his method of calibrating it. Kohlrausch (12) moreover, makes the categorical statement that all König's work was done with dark-adaptation.

In Abney's work the conditions of observation clearly require a dark room and therefore introduce dark-adaptation, although the fact is not made explicit and must be inferred from the nature of the apparatus. As appears from his use of a zinc oxide receiving surface for lights from both spectra and for white light, the room must have been secured against outside light. If it were not so, the stimuli could not have been controlled quantitatively.

In the more recent work of Wright a period of dark-adaptation of from 5 to 10 minutes was deliberately allowed, for the purpose of overcoming distractions and getting the subject settled.

The fact that in the work of König, Abney and Wright, the eye was dark-adapted, while in ours it was light-adapted should be weighed as partly accounting for the disagreement of our results with those of the others in regard to where the loss of saturation begins. It is also significant in explaining the physiological mechanism which determines the loss of saturation. This matter we shall now examine.

Laurens (7) in his work on the visibility curve of various animals and man, as evidenced by the pupillomotor effect, has shown that when the dark adapted eye is stimulated with weak light, the response of the pupil follows the scotopic visibility curve. When the light-adapted eye is stimulated with bright light, the pupillary response is identical with the

photopic visibility curve. Furthermore when the dark-adapted eye is stimulated with bright light, the response is intermediate.

This indicates that the rods contribute to the response of the dark-adapted-eye, even with stimulus intensities that are usually regarded as photopic. Now, since the stimuli which are used as the green primary are more effective in stimulating the rods than are the longer wave lengths, it is to be supposed that under conditions of dark-adaptation the red-green mixture would elicit more of the rod process than would the monochromatic intermediate light which is being matched and which is of longer wave length than the green primary (12). Since, however, rod stimulation produces a white process, excitation of the rods would tend to reduce the relative saturation of the field containing the shortest wave length, in proportion to its rod stimulating power. The field containing the shortest wave length is, of course, the mixed field; and the mixed field is the one which is reported to be of less saturation.

This argument depends upon the supposition that there are scotopic elements in those retinal areas which were used in the observations of König and the others who found extensive loss of saturation. In support of this still further, there is good evidence that the fovea is not entirely rod free (14, p. 56).

The stimulus intensity is important in relation to dark-adaptation. If the fields are illuminated to an intensity which is similar to or less than that of objects in the room to which the eye is already adapted, (the conditions of all ordinary useful vision), it is possible to match intermediate yellows with no evidence of loss of saturation in the dichromatic field, i.e., according to our present interpretation with no rod stimulation. If, on the other hand, the field intensity be greater than that to which the eye is adapted, the eye being relatively dark-adapted, the paleness of the mixed field becomes strikingly evident. Under these conditions the shorter component of the dichromatic field is noticeably effective in eliciting the achromatic rod process.

B. RELATION OF THE GREEN PRIMARY TO LOSS OF SATURATION

A wave length below 517 will, because it possesses blue stimulating value, introduce white and, therefore, desaturation, into a yellowish mixture of which it is a part. Hence, the loss of saturation in König's matches, aside from the part which is due to dark-adaptation, is in part due to a green primary of short wave length capable of also stimulating the blue sensory apparatus. We believe that in consequence of the careful choice of 517 for its "purity," and its consistent employment in all

matches, we have been able to get mixtures without a loss of saturation as far down as 517. This is represented in the color mixing curves of Fig. 2 by the termination of the blue curve at 517.

We have found it advantageous to compare our results with those of Wright (11) and of Weaver (10) in regard to saturation. Following a

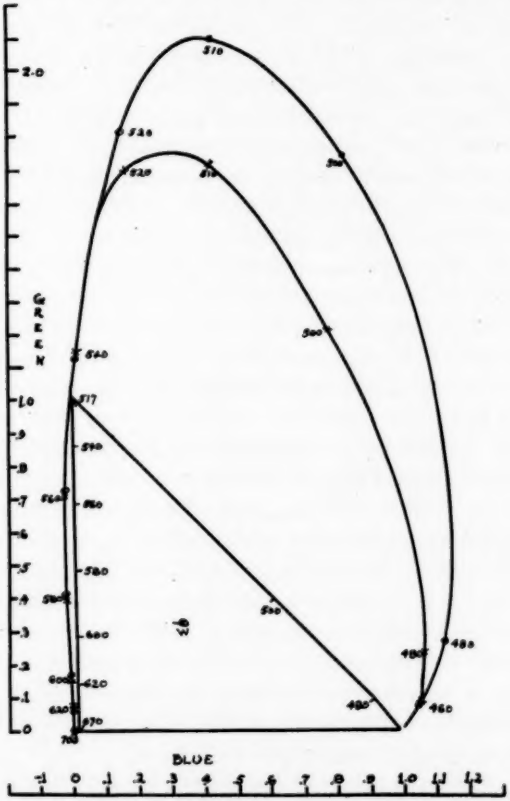

FIG. 3. *"Unit" equation color triangle.*
— *our data*
× *Wright's data*
0 *Weaver's (König's and Abney's) data*

method used by Ives (5) and Guild (3), Wright has made mathematical transformations which result in "unit" color mixing equations. These are then plotted on a right-angled triangle, in such a way that two of the primaries may be read off immediately as coördinates, and the

third derived by subtracting the sum of the two from unity. We have plotted Wright's values, which he has secured by a recalculation of his original data to conform with N. P. L. standard primaries (11), with green and blue as the coördinates. We have also plotted in the same diagram, Fig. 3, Weaver's values, which are a composite of the data of König and Abney, after they had been submitted to a similar recalculation in order to make them consistent with the N. P. L. primaries (10).

As our slit-widths could be used to get "unit" color mixing equations, we made the necessary calculations to secure values which could be plotted in the same system as those of Wright and Weaver above, so that all three sets might be considered analogous. Here we must call attention to the inconsistency with which we may be charged in plotting together sets of data which are based on different primaries. We chose not to render our data fully consistent with that of Weaver and Wright, and, hence, not to make a transformation based upon the N. P. L. primaries, because to have done so would have introduced a distortion of the facts. It would have thrown our 517 green primary off the apex of the triangle, which is where it belongs as a primary.[2]

Our data fall directly on the triangle everywhere except at the extreme blue. Wright's and Weaver's are outside (indicating negative values for certain primaries), and convex (indicating extensive desaturation). In the light of these facts we must seriously question the inherence which has been attributed to the loss of saturation by other workers.

We should venture to attribute the loss of saturation in the matches of König, Abney and Wright to dark-adaptation and to the use of a green

[2] We feel obliged to note that in making the mathematical transformations of our data into the "unit" equations of Ives, Guild, and Wright, we have not conceded the propriety of such a procedure. (We hope in a future paper to be able to demonstrate that the physiological realities are not represented with equal fidelity by all the theoretically equivalent results which are produced by these transformations.) We regard this transformation of our data as an at present unavoidable expedient in order to supply a common basis of comparison for the three sets of data. As our comparison involves only the question of saturation, and as the right-angled presentation symbolizes the relationship in this regard between the three sets of data, we feel justified in using this form so long as we do not commit ourselves to the more general principal that transformations give physiologically equivalent values. We feel that it is in order to question the propriety of these transformations and "unit" equations, frequently involving negative quantities which can have no direct and immediate physiological significance. The results of such operations represent relationships which are so many removes from the actual data and the physiological processes themselves that they would seem to obscure the picture of physiological events in the visual mechanism that we are primarily interested in getting.

primary other than 517. Such an arbitrary primary other than 517 would stimulate, in addition to green proper, red or blue processes accordingly as it was longer or shorter than 517. The supplementary process of the green primary together with a small amount of the processes excited by the other stimulus would thus produce an appreciable amount of desaturating white in the mixture. It is difficult to say with assurance in the case of any one of these workers, which of the two factors predominated in causing the desaturation.

C. RELATION OF YELLOW TO DESATURATION

The blue curve has undergone an extension above 517 for still another reason, the implications of which are so important, that we shall now examine it in detail.

Hecht (4) has developed, from the hue discrimination data of Laurens and Hamilton (8), a set of three curves which are tentatively suggested as an embodiment of the spectral characteristics of the three color receptors. These curves are coextensive in that the blue curve reaches to the red end of the spectrum and the red curve clear down into the blue. They have been drawn in such a way that by apparently self-consistent manipulations they may be made to yield the normal hue discrimination data, the Grundempfindungen of König as recalculated by Weaver, and the standard visibility curve.

It is difficult to see how a set of curves with the blue primary extending up into the red can be made to harmonize with the color mixture data which we present. It is possible, however. Suppose the ordinates of the blue curve to be so drawn that they shall always be a certain proportion of the ordinates of the green at all wave lengths, including and above 517. Suppose further that these curves begin to approach each other at about 510 and cross at 490. One would then have a set of curves which would fit our data and be minimally changed from Hecht's original curves. Curves constructed in this fashion would differ in their implications from those which we have presented in Fig. 2, in that the Hecht type would postulate large amounts of all three primaries in the orange-yellow-green parts of the spectrum, whereas ours of Fig. 2 would postulate only two.

The most evident reason why Hecht put the peak of the blue curve in the yellow is apparently that yellow is conventionally regarded as the least saturated part of the spectrum, and because traditionally unsaturation is postulated as resulting from the ability of a stimulus to

excite red, green, and blue receptors simultaneously. This classical postulate derives its force from frequent repetition. It causes confusion because it is unwittingly extended beyond the limits to which it is logically bound. The argument for the extension of the blue curve, if it were formulated, may be supposed to be somewhat as follows.

Since achromatic white may be produced by mixing stimuli which separately give the chromatic sensations of red, green and blue, it is to be assumed that each chromatic stimulus in this mixture has equal power in cancelling the chroma of the other two. This assumption of equal chroma cancelling power—unbased in experiment—has been formalized in the color equations. The symbols used in this formalization—now inadvertently shorn of the qualifications which, from their derivation, should be attached to their meaning—have been bodily transposed into situations where by unconscious implication they have been given an entirely new and illegitimate meaning. Thus, the equation $R+G+B=W$, has by the assumption of equal chroma cancelling power, been given the form $R=G=B$ in white, following, of course, the best fashion in trichromatic theory.

This statement of assumed equal chroma cancelling power has been used by Hecht and others, to support the postulate that whiteness involves the equal brightness (luminosity) of the sensation elicited by the stimulation of each of the three receptors (4, p. 245, p. 246).

On a similar basis Sinden (9) implies that yellow stimuli elicit a response of the blue receptor. Since the complementary to yellow sufficiently intense to cancel the yellow chroma is much less luminous than the yellow itself, it must, therefore, be that the yellow contains blue; i.e., is unsaturated in the usual sense, if we are to go by the traditional formula $R=G=B$ in white, and extend (erroneously) the ideas of chroma cancelling power to the luminosity situation.

The Gordian knot may be cut by discarding the traditional definition of white and assuming that red and green, when mixed alone, tend to cancel each other's chroma without the intervention of any blue process, and the phenomena (Sinden (9) and Priest and Brickwedde (13)) which have led Hecht to postulate a blue peak in the yellow are simply the result of the above described pseudo unsaturation of yellow.

Aside from these considerations there is another reason why we have not favored curves of the modified Hecht type. It is to be found in an analysis of the color system of the two types of dichromats. One of the essential peculiarities of both the protanopic and deuteranopic eye (Laurens and Hamilton (8) and unpublished data) is that hue discrimi-

nation is lacking above the wave length 517.[3] This fact could be represented as far as the protanope is concerned, by modifying the Hecht curves as described in an earlier paragraph; i.e., making the blue curve follow the protanopic "green" curve at a height that is always proportional to their relative heights at 517. Since the deuteranope eye is also unable to perceive color differences above 517, one would have to express this, in the Hecht type of diagram, by assuming an altogether different blue curve; i.e., one whose ordinates are proportional to the ordinates of his "long" curve in the same fashion as the first described blue curve is proportional to the ordinates of the protanopic "green" curve.

This necessity of having two blue curves, one for the protanope and one for the deuteranope, would make the problem of realistically analyzing the mechanism of color vision very complicated. It would also destroy the (possibly exaggerated) reference value of the dichromatic system in analyzing trichromatic vision. Further, it would introduce so many qualifications into the basic theory that the possibilities of internal inconsistency, when the theory is applied to new data, would be greatly increased. The difficulty is eliminated by assuming that the blue receptors are not stimulated by wave lengths above 517, and by not assuming that the low saturation in the yellow is due to trichromatic stimulation.

7. PECULIARITIES OF MIXTURES BELOW 517

Since under the conditions of our experiments, the red-green mixtures are quite as saturated to the light-adapted eye as are the monochromes which they match, the simplest assumption is that the retinal elements which elicit the blue sensation are not stimulated at all by the light of greater wave length than 517.

Mixtures of 517 and the various blues or violets, however, are less saturated than their monochromatic equivalents. This loss of saturation begins with matches in the vicinity of 500 and increases toward the shorter wave lengths. It seemed convenient to reduce this desaturation to a minimum. We, therefore, selected 480 as a primary. This wave length obviously stimulates the green perceiving apparatus, but the

[3] In making color mixture matches, 517 without the addition of blue would serve to make a perfect match with all wave lengths greater than 517. That this is so with the protanope may signify the loss of a process which corresponds to the red apparatus in the normal. The fact that the deuteranope can also make the same matches implies that the remaining functional process is very similar to the normal green curve and to the protanopic "long" curve in the middle regions of the spectrum. The principal difference seems to be that it extends much farther into the red.

amount of such stimulation can be evaluated as we shall see in the next paragraph. Now mixtures of 480 and 517 match intermediate wave lengths with very little loss of saturation. The reason which we would propose for this is that 480 stimulates the red perceiving apparatus much less than does 460 or shorter wave lengths.[4] It was inferred that it is this reddishness which combines with the blue proper of the short primary and some of the green of 517 to form white. This white accounts for the loss of saturation in the mixed field as compared with the monochromatic field and is much less with 480 as a primary than with 460 or lights of shorter wave length. This loss of saturation is evident but not measureable by the method which will be described for 460. In fact it is so small that it is overcome at once if the eye is fatigued to red.

Now if a subject is asked to match white with the three primaries 670, 517 and 480, and then 460 is added to the standard white, he will increase the amount of 670 and of 480, but decrease the amount of 517, before he accepts the match. On the assumption that 460 does not stimulate the green apparatus, and that 517 does not stimulate either the blue or the red apparatus, we are left with the conclusion that 480 stimulates the green to the extent that 517 is taken out of the trichromatic match for white plus 460, and that 460 stimulates the red to the extent that 670 is added to the mixture which is adjusted to match white plus 460. The figures came out so that 0.017 of 460 was red and 0.095 of 480 was green.[5] The sensitivity of the blue apparatus was plotted accordingly; e.g., as Table 5 shows, at 460 0.000 mm of slit-width is substituted for -0.040 of green (this substitution is an increase of the green and an elevation of the curve at this point), and 0.383 mm is substituted for the 0.423 of blue (this substitution is a decrease of the blue to equal the above increase of the green and is represented by a corresponding depression of the blue curve at this point). For matches to 480 and 500 exactly similar adjustments in Table 5 and Fig. 2 have been made.[6] As a result of this systematic depression of the blue curve and the corre-

[4] It is, of course, conceivable that the red impurity is in the 517 and not the 480. In the discussion of this very minute amount, which we have been unable to measure, we have arbitrarily ascribed it to 480, and in our plots ignored it in dealing with mixtures to match 500.

[5] For 500 down the elevation of the green curve is justified because it is based directly upon measurements of the slit of the green primary. The corresponding depression of the blue curve has arbitrarily been made equal to the elevation of the green. This is consistent because all data have been presented in terms of slit widths without regard to chroma cancelling power and luminosity.

[6] For the same reason similar adjustments were made in Table V for the values of green and blue in the white match, the green designation being increased and the blue decreased.

sponding slight elevation of the green in matches of wave lengths 500 and less, we have succeeded in giving a tentative picture of the processes in the visual mechanism. Because the blue curve as drawn does not represent directly the slit-width readings, as does the 670 for instance, we have labelled it "Blue" instead of 480. Accordingly the curve is to be interpreted as representing the amount of blue physiological process excited. This procedure, undertaken purely as a tentative step and nowise affecting the other properties of the curves, seems to be helpful in avoiding the physiological incongruity of a negative green process in the visual mechanism.

From 500 to 460, the green curve has been elevated slightly in the manner already described. It, like the blue, does not represent the primary directly, and should in this region have been labelled simply "Green" to correspond with the "Blue." But since as far down as 500 it does represent the primary 517 directly, we thought it less confusing if we avoided an additional label in the curve and accounted for the discrepancy in labelling in the text.

In the match to 460 it was possible to measure the amount of 670 which had to be added. It gave a value for the red excitation produced by 460, of 0.007 mm of slit-width, as indicated in Table 5 and Fig. 2. As the amount of this red was too minute to be determined for 480 and as it presumably reached *O* in this region, no deviation from the facts occurs in plotting it as running from *O* at 480 to 0.007 at 460. This red component we have labelled "Red" in Fig. 2.

8. SUMMARY

A group of 68 subjects has been studied in order to make quantitative determinations of the trichromatic functions of the average normal eye.

Tables 1–4, inclusive, show that in a mixture to match white or a monochrome the sensitivity of the population to a given primary follows the normal distribution curve, and not a bimodal segregation as König has supposed. The variations in sensitivity in the population are primarily due to variations, from person to person, in receptor functions, and secondarily perhaps to variations in macular pigmentation.

The loss of saturation in mixtures which match monochromes from 577 to 517 (Maxwell, König, Abney and later workers) is due first to the employment of a dark-adapted retina; and secondly to the employment of an "impure" green primary. In our work the retina was adapted to an intensity which is similar to the intensity of the photometric fields. This situation involves the conditions of all ordinary useful vision. The stimulation did not overstep the fovea. Desaturating scotopic factors

were thus eliminated. Wave length 517 was employed as the green primary for its relative freedom from the red and blue stimulating capacity which would have a desaturating effect in mixtures. Consequently, as Fig. 2 shows, under these conditions, the blue excitation curve may not be extended above 517; and saturated matches may be produced for the entire visible spectrum, except below 500. The fact that stimuli of short wave lengths excite the red as well as the blue process may account for unsaturated mixtures in this part of the spectrum.

The paleness of the yellow *monochromes* has been attributed to trichromatic desaturation, involving varying proportions of the blue process (König, Abney, Hecht, etc.). The fallacious argument for this rests upon the unwitting transfer of symbols developed in the analysis of chromatic relationships, to situations involving luminosity.

BIBLIOGRAPHY

1. Abney, Sir W. De W., *Researches in Color Vision*, London, 1913.
2. Forsythe, W. E., Nela Park, Ohio, personal communication.
3. Guild, J., *The transformation of trichromatic mixture data; algebraic methods*, Trans. Opt. Soc., London, *26*, 95; 1924–25; *The geometrical solution of color-mixture problems*, Trans. Opt. Soc., London, *26*, 139; 1924–25.
4. Hecht, S., *The development of Thomas Young's theory of color vision*, J.O.S.A., *20*, 231; 1930.
5. Ives, H. E., *The transformation of trichromatic mixture equations from one system to another*, Frank. Inst. J., *180*, 673; 1915.
6. König, A., Gesammelte Abhandlungen zur physiologischen Optik, Leipzig, 1903, 214. *Die Grundempfindungen in normalen und anomalen Farbensystemen und ihre Intensitätsvertheilung im Spectrum.*
7. Laurens, H., III *The pupillomotor effects of wave-lengths of equal energy content*, Amer. Jour. Physiol. *64*, 97; 1923.
8. Laurens H. and Hamilton, W. F., *The sensibility of the eye to differences in wave-length*, Amer. Jour. Physiol., *65*, 547; 1923.
9. Sinden, R. H., *Studies based on spectral complementaries*, J.O.S.A., *7*, 1123; 1923.
10. Report of Colorimetry Committee of the Optical Society of America, J.O.S.A., *6*, 527; 1922.
11. Wright, W. D., *A re-determination of the trichromatic coefficients of the spectral colors*, Trans. Opt. Soc., London, *30*, 141; 1928–29; *A re-determination of the mixture curves of the spectrum*, Trans. Opt. Soc., London, *31*, 201; 1929–30.
12. Kohlrausch, A., *Tagessehen, Dämmersehen, Adaptation*, in Handbuch der Normalen u. Pathologischen Physiologie, 12, 2, 1534, Berlin, 1931.
13. Priest and Brickwedde, J.O.S.A., *13*, 306; 1926.
14. Parsons, Sir John Herbert, *Color vision*, Cambridge, 1924.
15. Franceschetti, A., *Die Bedeutung der Einstellungsbreite am Anomaloskop etc.*, Schweitz Med. Woch., *58*, 1273; 1928.
16. Planta, P., *Die Häufigkeit der angeborenen Farbensinnstörung etc.*, Graefe's Arch. f. Ophthalmol., *120*, 253; 1928.
17. Collins, M., *The Rayleigh Color Equation with Rotating Disks*, Brit. Jour. Psychol., *19*, 387; 1929.
18. Schuster, A., *Experiments with Lord Rayleigh's Colour Box*, Proc. Roy. Soc., *48*, 140; 1890.
19. Houstonn, R. A., *A Statistical Survey of Colour Vision*, Proc. Roy. Soc., A *94*, 576; 1918.

Wave-length Tables for Spectrum Analysis. By F. Twyman and D. M. Smith. 192 pages. Adam Hilger, London. 14s 9d post free.

A "fully revised and much enlarged" second edition of a set of tables first published in 1923, this present edition contains much useful information in a condensed, well selected, and readily usable form. As the title implies the main objective has been to furnish information useful in connection with the usual procedures of making spectrum analyses for chemical and metallurgical purposes. Unlike the previous edition, the I. A. scale of wave lengths has been used throughout. The most recently adopted interferometric measurements of secondary wave-length standards are included in full. Among other items of information that the analytical spectroscopist will have frequent occasion to use are to be found tables of distinctive lines in the spark spectra of solutions and solids, lists of sensitive arc lines of some fifty elements and of certain sensitive lines in flame spectra, and a classification of ultimate lines (sometimes identical with raies ultimes) of about sixty elements. Each table is accompanied by brief but pertinent explanations and discussions. In an appendix is given a brief account of results obtained from a condensed spark with comments on the effects of varying the capacity and self-induction. The methods of exploded wires and high frequency sparks are briefly mentioned. Although all of the data given are "entirely verbatim transcriptions," references to the original sources drawn upon are given. Within the intended scope of the compilations the selection of material is highly satisfactory. As a book for ready reference there is much to commend it and many spectroscopic laboratories will find it indispensable.

R. C. GIBBS

Photoelectric Phenomena. By Arthur Llewelyn Hughes and Lee Alvin DuBridge. First Edition, 1932. Pp. xii+531. McGraw-Hill Book Co., Inc., New York. Price $5.00.

This volume is another in the International Series in Physics, F. K. Richtmyer, Consulting Editor. It serves the same general purposes as do the previously published volumes of the series, namely, to furnish a survey of the field to those who wish to gain a general notion of photoelectric phenomena and to give to the workers in that field a readily accessible summary of the work to date, including both theory and experiment. As the authors point out, the book will serve also to give the fundamental principles to those engineers who are working in the field of practical applications of photoelectric effects.

The first chapter is a brief introduction in which the authors outline the field to be covered in the book. This field is divided into three divisions as far as the principal subject matter is concerned, namely, Surface

Photoelectric Effects, Volume Photoelectric Effects and Photovoltaic effects. A list of books on the general subject is appended to this chapter.

The next four chapters are devoted to the descriptions of experimental results in the division of Surface Photoelectric Effects. The chapter headings will give one a notion of the topics treated. They are: The Fundamental Laws of Photoelectricity, The Photoelectric Threshold, Energy and Spectral Distribution of Photoelectrons and the Selective Photoelectric Effect. The descriptions of the experimental results are critical and very often furnish suggestions as to further work to be done. Although the theories of photoelectric emission are treated in a separate chapter, references to the theory are made in connection with the descriptions of the experiments so that the mutual relations of experiment and theory are made clear. There are several tables in these chapters which will be of value to workers in the field as they are compilations from a very extended literature and contain the best experimental results to date. These tables are headed: Photoelectric and Thermionic Work Functions of the Metals, Constants in "Complete" Photoelectric Emission, Effect of Gases on the Photoelectric Emission of the Metals and Position of Maximum Spectral Selectivity. The exhaustive list of references given in connection with the text of the book will make it a valuable volume for the research room.

The chapter on the Theories of Photoelectric Emission contains a survey of the older theories and their successes and limitations are pointed out. The newer theories based on the Sommerfeld electron theory of metals are discussed at some length and their coordination with recent experimental work shown. As an indication of the up to date character of the book, one finds a discussion of the Fowler method of analysis together with descriptions of the very recent experiments on the effect of temperature on the photoelectric emission for frequencies near the threshold value. The connection of thermionic emission and the photoelectric emission is brought out in the light of the newer theories.

The next five chapters are concerned with the experimental results in the fields of: Ionization of Gases and Vapors by Ultraviolet light, Photoconductivity, Photovoltaic effects, Photoelectric Effects in Non-Metallic Substances and Photoelectric Effects of X-Rays and γ-Rays. In these chapters the authors point out that the recent theories make a more intimate connection between these effects and the surface photoelectric effect than had been accomplished by the older theories.

Chapter XII is of a kind which is unfortunately omitted in some treatises of this kind, but which makes such a book much more valuable to those who wish to begin work in the field. It is entitled Photoelectric Technique and gives information on the subjects of design of cells, methods of preparing the active surfaces, vacuum technique, methods of current measurement, sources of light, and methods of measuring spectral energies. The

tables on Sources of Monochromatic Light and on Short Wave Cut-Off Filters will be of value to workers in other fields than photoelectricity.

The last two chapters are brief ones on Applications of the Photoelectric Effect and on Miscellaneous Phenomena. This latter chapter is concerned with experimental results which are, as yet, not readily coordinated with the principal effect, but which are apparently worth while mentioning. The appendix contains a few tables of constants and a periodic chart of the elements. The index includes a name index as well as a subject index.

A very few minor misprints were noted which are for the most part obvious to the casual reader and do not detract from the value of the book. The book will be welcomed by all those for whom it was written.

J. R. COLLINS

Science in Action. By Edward R. Weidlein and William A. Hamor, Mellon Institute of Industrial Research. McGraw Hill Book Company, 1931; 310 pages, 30 illustrations.

A book with this title written by such prominent authors needs no recommendation. Technology has been proven without question the most important factor in the rapid development of our nation. Its possibilities were appreciated by George Washington who in his first annual address in 1790 stated "There is nothing which can better deserve your patronage than the promotion of science." Development of this spirit, so typical of our early statesmen, our Franklins and our Jeffersons, has accelerated until today practically every moment of our lives is influenced by social and economic forces, the origin of which can be directly traced to the chemical, physical and engineering research laboratory. The subject is therefore one concerning which volumes could be effectively written.

Chapter I entitled the Groundwork of Industrial Research is introductory. Chapter II describes the Development of Industrial Research in the United States. Some readers no doubt will be surprised by the vision of many of the industrialists of a century ago in their attitude toward research. Chapter III, The Present Status of Industrial Research reviews the general field of investigation. This subject is elaborated upon in Chapter IV entitled Types of Industrial-Research Laboratories, such as privately promoted, sponsored by industrial and trade organizations, fellowships, etc. Chapters V and VI deal with the relation of science to human welfare, problems concerning air pollution, ventilation, smoke and dust, sleeping equipment, water purification and analysis, preservation and inspection of food products, sanitation, prevention and control of disease, pharmacy, effects of science upon mortality.

Chapter VII is devoted to a brief description of Success of Scientific Research in Familiar Industries. Examples are, agricultural machinery, fer-

tilizer, insecticides, cottonseed oil, corn products, sugar, textiles, dyes, synthetic fibers, leather, petroleum, electrical industries, telephone, radio, aviation. Chapter VIII discusses Some Scientific Research Contributions to Engineering Economics, more specifically, alloy steels, by-product coke, cement, pavement, refrigeration, fuels, heat insulation. The following chapter summarizes the experience of various organizations in the application of research to the reduction of waste products. Chapter X discusses plastics; rubber, pyroxylin, photographic materials, viscose, synthetic resins and casein. The next chapter describes some of the achievements of chemical syntheses, products of natural gas, ammonia, ethylene glycol, perfume. Some 23 pages are devoted to Scientific Research on Problems of Industrial Management. The title of Chapter XIII is Industrial Research, Machines and Labor. This 14 page monograph could well be expanded into a book by itself. As it stands, it should furnish food for thought for economists who are still laboring under the burden of theories applicable a century or two ago, little appreciating that even as they write, some profound social or economic principle is being completely recast in the laboratory of the humble chemist, physicist, or engineer. In this chapter the myth of technologic unemployment is exploded. The only hope for employment is through the sciences which in this country alone can be responsible for 4,300,000 employees in the automobile industry. Unless we return to the medieval agricultural age, where women harvested the crops and everyone was employed 14 hours a day in order to exist, we must ground our faith in science which is able to create new desires in men and new industries, such as the telephone, aviation, refrigeration, cinematograph, and countless others. Labor saving devices and industrial research bring new opportunities to the working class, a fact which is substantially recognized in a resolution by the American Federation of Labor.

Chapter XIV discusses Scientific Research, Management and Banking and quite properly intimates that the stocks of certain large corporations are no better than the research departments which the corporations maintain. The final three chapters are entitled The International Aspects of Industrial Research, Industrial Research Laboratory Management, and Industrial Research Workers. The book concludes with extensive indexes, both subject and name.

We need more monographs of this type. The scientist has a mission in life and a story which is worth relating. Let him come out of his cloister, play the active rôle in social and economic life, and assume the responsibilities which he alone is able to shoulder. The modest attitude that science can make its own way without publicity is passé in the modern rush of life. Even the good influence of religion requires earnest propaganda by several thousand ministers, priests and rabbis in America. Surely there must be physicists and engineers who can carry on the work started by the chemist-

authors of this book. Those who feel incompetent to take part in such a program can at least purchase a copy of Science in Action, thereby encouraging the missionary enterprise, and at the same time increasing the value of their library by a book which is readable, authoritative and inspiring.

PAUL D. FOOTE

Slit-Lamp Microscopy of the Living Eye. Early Diagnosis and Symptomatology of Affections of the Anterior Segment of the Eye. By Dr.F. Ed. Koby (Basle), Translated by Charles Goulden and Clara Lomas Harris. Second Edition, Blakiston, Philadelphia, 1930.

This book is designed to provide ophthalmologists with a convenient handbook to assist them in learning to apply this relatively new method of diagnosis. It is frankly not written for the beginner and even the ophthalmologist of some experience will find it desirable to acquaint himself in an elementary way with the subject in general before reading it.

The first 34 pages deal with apparatus. Then follow chapters on methods of examination and on phenomena of reflection of light. The remainder of the book deals with the appearances which may be observed by this method in the various eye structures which are accessible for observation and with their interpretation. Twelve pages are devoted to the conjunctiva, seventy-eight to the cornea, fifteen to the anterior chamber, thirty-six to the iris, seventy-nine to the crystalline lens and twenty-two to the vitreous body. There are 104 illustrations. The text is followed by twenty-two pages of bibliography, a most useful feature in a work dealing with a subject of recent origin in which ideas are not as yet fully crystallized. There is also a very complete index.

On page 277 occurs the curious statement, "Freese examined a woman of forty-eight years four months after electrocution." It seems likely that the word which has been rendered "electrocution" really meant electric shock.

This work should be of great value in promoting a more general understanding and an increasing use of a method of ophthalmological examination which promises in some conditions an earlier diagnosis and in many a more complete knowledge of the situation.

H. B. WILLIAMS